Comment on "Trapping force, force constant, and potential depths for dielectric spheres in the presence of spherical aberrations"

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I point out a confusion that is rather common in optical forces, i.e., that the time average of the Lorentz force on a dipole (for a harmonic time-varying field) is sometimes assumed to be a gradient force that is due to omission of the radiative reaction term in the polarizability of the dipole.

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1. Introduction

In a recent paper Rohrbach and Stelzer[1] derived the optical force acting on a lossless dielectric particle illuminated by a harmonic time-varying field. The authors started by calculating the electromagnetic density force (i.e., the force per unit volume) $\mathbf{f}(\mathbf{r}, t)$ acting on a small polarizable particle with dipole moment $\mathbf{p}(\mathbf{r}, t)$. The density force was written as (for more details of their method, see Ref. 2)

$$\mathbf{f}(\mathbf{r}, t) = [\mathbf{p}(\mathbf{r}, t)\nabla]\mathbf{E}(\mathbf{r}, t) + \frac{\partial \mathbf{p}(\mathbf{r}, t)}{\partial t}\mathbf{B}(\mathbf{r}, t). \quad (1)$$

This is the Lorentz force. Using the relations $\mathbf{p}(\mathbf{r}, t) = \alpha\mathbf{E}(\mathbf{r}, t)$ and $\nabla\mathbf{E}(\mathbf{r}, t) = -[\partial\mathbf{B}(\mathbf{r}, t)/\partial t]$, they obtained

$$\mathbf{f}(\mathbf{r}, t) = \nabla[\alpha\mathbf{E}(\mathbf{r}, t)\mathbf{E}(\mathbf{r}, t)]/2 + \alpha\frac{\partial}{\partial t}[\mathbf{E}(\mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t)], \quad (2)$$

which is the total force experienced by the particle. Notice that α is the polarizability of the particle and that $\mathbf{m}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{B}(\mathbf{r}, t)$ is proportional to the Poynting vector. At optical frequencies one needs to work with the time average of Eq. (2). The first error made by those authors was to interpret $\langle\partial\mathbf{m}(\mathbf{r}, t)/\partial t\rangle$ as the difference between the time-averaged modulus of the Poynting vector scattered by the particle and the incident Poynting vector; they wrote

$$\mathbf{f}(\mathbf{r}) = \langle\mathbf{f}(\mathbf{r}, t)\rangle = \frac{\alpha}{2c}\nabla I(\mathbf{r}) + \alpha\frac{\langle\mathbf{m}(\mathbf{r}, t_{\text{after}})\rangle - \langle\mathbf{m}(\mathbf{r}, t_{\text{before}})\rangle}{\Delta t}, \quad (3)$$

where the subscripts "before" and "after" refer to times before and after scattering, respectively. Note that the first term in Eq. (3) is the gradient force; after some tedious calculations Rohrbach and Stelzer showed that the second term is the scattering force. They supported their reasoning by citing the work of Gordon.[3] However, Gordon used this reasoning for a pulse, and he emphasized that, for a standing wave, $\langle\mathbf{m}(\mathbf{r}, t)\rangle$ vanishes. This is a well-known fact that has been pointed out elsewhere (see, for example, Ref. 4). Only when the intensity of the field is modulated at a low frequency can one hope to measure the optical force produced by this term.[4–6] Unlike for a pulse, for which a harmonic field is assumed, there is no after or before. In that case the scattering force may be derived from an erroneous computation. In fact, their assumption is that the difference between the momentum carried by the scattered light and that by the incident light is directly related to the second law of Newton and hence gives the scattering force. In fact, for a standing wave the total time-averaged force is given by the time average of the first term of Eq. (2) as the second term vanishes:

$$\mathbf{f}(\mathbf{r}) = (1/2)\langle\nabla[\alpha\mathbf{E}(\mathbf{r}, t)\mathbf{E}(\mathbf{r}, t)]\rangle. \quad (4)$$

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This expression seems far from that of a total force acting on a small polarizable particle, as the scattering force does not appear explicitly. The second confusion of the authors of Ref. 1 is rather common, as they wrote that the time average of Eq. (4) is the gradient force, i.e., $\alpha\nabla I(\mathbf{r})$. This is no longer the case. The mistake lies in assuming that $\mathbf{p}(\mathbf{r}, t) = \alpha_0\mathbf{E}(\mathbf{r}, t)$, where α_0 is related to the Clausius–Mossotti relation, which for a lossless material yields real polarizability.[1,2] In fact, one must not forget that the total field at the position of a polarizable particle is the sum of incident field $\mathbf{E}(\mathbf{r}, t)$ and the field that is due to the particle at its own location, $\mathbf{E}_s(\mathbf{r}, t)$ (i.e., the radiative reaction term).[7] For small polarizable particle, this radiation-reaction field can be written as[7,8]

$$\mathbf{E}_s(\mathbf{r}, t) = i(2/3)k^3\mathbf{p}(\mathbf{r}, t), \tag{5}$$

where k is the modulus of the wave vector. Therefore the correct dipole moment for a small polarizable particle is given by[9]

$$\mathbf{p}(\mathbf{r}, t) = \alpha\mathbf{E}(\mathbf{r}, t) = \alpha_0[\mathbf{E}(\mathbf{r}, t) + \mathbf{E}_s(\mathbf{r}, t)], \tag{6}$$

which gives the following well-known form for the polarizability[9]:

$$\alpha = \alpha_0/[1 - (2/3)ik^3\alpha_0]. \tag{7}$$

It is important to make the correction to the Clausius–Mossotti relation to satisfy the optical theorem and derive the correct expression of the optical force.[9] The net force, from Eq. (4), is then given by[10,11]

$$f_i(\mathbf{r}) = (1/2)\mathrm{Re}\{\alpha E_j(\mathbf{r})\partial_i[E^j(\mathbf{r})]^*\}, \tag{8}$$

which contains the gradient and the scattering force. For example, if we compute the net force on a minute sphere, using Eq. (8), when the incident wave is a plane wave $[E_x = E_0 \exp(ikz)]$, we find that $f_z = k^4\alpha_0{}^2|E_0|^2/3$, which is the scattering force for a small sphere.

In conclusion, the expression used by Rohrbach and Stelzer to compute the optical force on their object, although it led to the correct result (the gradient force plus the scattering force), because of two mistakes that compensate for each other is based on flawed reasoning. First, there is no branching of the interaction for harmonic fields and hence there can be no splitting of the interaction into before and after events; hence the time average of the Poynting vector vanishes. Second, it is essential to include radiation reaction to satisfy the law of energy conservation. Equation (4), which is the total force, gives only the gradient force: The first error, which gives the scattering force, compensates for the omission of the scattering force in Eq. (4).

References and Notes

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