An enhanced contrast to detect bulk objects under arbitrary rough surfaces

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Abstract: We study a selective light scattering elimination procedure in the case of highly scattering rough surfaces. Contrary to the case of low scattering levels, the elimination parameters are shown to depend on the sample microstructure and to present rapid variations with the scattering angle. On the other hand, when the slope of the surface is moderated, we show that these parameters present smoother variations and little dependence to the microstructure, even when the roughness is high. These results allow an important selective reduction of the scattered light, with a basic experimental mounting and an analytical determination of the elimination parameters. Such selective scattering reduction is demonstrated by simulations and experiments and applied to the imaging of an object situated under a highly rough surface.

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References and links

1. Introduction
Wave detection of a buried object beneath a rough surface still remains a challenge for numerous applications [1, 2] including land-mines detection. In a similar way laudable efforts have been put on optical microscopy [3] to reach enhanced contrasts able to emphasize the presence of single ultra-thin biological objects. Also, new techniques have been developed in the field of imaging in complex media, in particular for biophotonics and biomedical applications [4, 5]. All these techniques may address near or far field regions, linear and non linear effects.

Within this framework our motivation was to elaborate a specific far field procedure allowing the identification of scattering emitted by surface roughness or bulk heterogeneities [6]. Experimental data first showed successful signatures [7] allowing to discriminate between surface and bulk samples of similar intensity patterns, which was made possible thanks to the development of a specific angle-resolved ellipsometer. With this set-up, ellipsometric parameters were measured for scattered laser light at each direction of space and the data highlighted fully polarized speckle patterns for low receiver solid angles. These results led us to try a general procedure [8] to cancel polarized light scattering in a selective way (surface versus bulk). The technique is elementary and uses a retardation device and an analyzer plate that are tuned at each direction of space according to the ellipsometric properties of the scattered field, to selectively eliminate the total field or one component of it. The procedure was shown successful at low scattering levels (the case of slightly rough surfaces or inhomogeneous bulks) since several experiments confirmed the strong reduction of surface scattering to the profit of bulk scattering, and inversely [9, 10]. At these low-scattering levels, all scattered fields are proportional to the Fourier Transform of defects (roughness or inhomogeneities), with the result that the ellipsometric parameters do not depend on the microstructure of the samples, but only on the origin of scattering (surface or bulk). In other words, light scattering from a surface or bulk can be canceled without
any knowledge of the sample topography or inhomogeneity. Such property explains the success of the selective scattering reduction procedure in the framework of perturbative theories.

Now a question concerns the generalization of the procedure to the case of arbitrary complex media. The situation is strongly different at least for 3 reasons:

• Rigorous electromagnetic theories must be used and they predict a strong dependence of the ellipsometric parameters versus the sample microstructure. Therefore the microstructure must be known to match the optical plates for scattering reduction, which a priori shows less interest. Though experiment allows to scan the scattering reduction parameters, the 2 dimensional scan (retardation phase and analyzer angle) will only be useful for total reduction (or cancellation), not for selective reduction (except if all objects under probing are known).

• High scattering samples exhibit high variations of ellipsometric parameters versus scattering direction, which makes the experimental procedure much more complex. Indeed in some situations the parameters should be matched every $10^{-2}$ degree, depending on the sample microstructure (roughness, inhomogeneity and associated correlation lengths or slopes). One solution may be found in using pixelized liquid crystals, but the resulting procedure becomes time-consuming and looses its simplicity.

• Even if the incident source is perfectly monochromatic, partial polarization may occur due to the high derivative of phase terms versus scattering angle inside the receiver solid angle. This third point was previously addressed in ref. [11] and was shown to be solved when receiver solid angles are decreased.

In the present work we address the generalization of the scattering reduction procedure to arbitrary rough interfaces and propose solutions to reduce these difficulties. The first step is to rely on a rigorous theory and analyze the ellipsometric parameters in the far field pattern. To reach this goal we have developed a computer code based on the differential method [12] including S-matrix algorithm [13] and fast factorization formulation [14]. Though the absolute phase and modulus of the scattered waves strongly depend on microstructure, we will see that, for most samples, the ellipsometric parameters (modulus ratio and phase difference) show little angular variations and little dependence on the microstructure. Moreover, in order to avoid systematic use of rigorous theories that are highly time-consuming for 3-dimensional cases, we compare the results to those obtained with approximate theories (first-order approximation [15, 16] and physical optics approximation [17]). Then the approximate scattering reduction parameters, that no more depend on the microstructure, are applied to reduce the light scattered by a very rough interface and to enhance the contrast of an object situated underneath. Both a simulation of the procedure and its experimental realization give very satisfactory results.

2. Principles of the technique

We consider a sample lighten by monochromatic and spatially coherent light (Fig. 1). The scattered field $E$ with wave vector $k$ can be written

$$E = (A_S + A_P) \exp(i k \cdot \rho),$$

(1)

with $\rho = (x,y,z)$ and $A_S$ and $A_P$ the components along the $s$ and $p$ polarization directions (Fig. 1), which can be written

$$A_S = |A_s| \exp(i\delta_s) e_s,$$

(2)

$$A_P = |A_p| \exp(i\delta_p) e_p,$$

(3)
with \( \mathbf{e}_s \) and \( \mathbf{e}_p \) the unit vectors along the directions \( s \) and \( p \). We note \( \delta \) the phase difference between \( p \) and \( s \),

\[
\delta = \delta_p - \delta_s \quad (4)
\]

If a tunable retarder (e.g. liquid crystal device or Pockels cell) with axes parallel to \( \mathbf{e}_s \) and \( \mathbf{e}_p \), inducing a phase delay \( \delta^* = \delta^*_p - \delta^*_s \), and an analyser making the angle \( \psi^* \) with the \( s \) direction, are introduced in the mounting (Fig. 2), the projection of the field on the analyzer direction is

\[
A' = \cos \psi^* \left( A_s + \tan \psi^* A_p \exp (i\delta^*) \right). \quad (5)
\]

Notice that the phase delay can also be induced by a quarter wave plate correctly orientated or by a tunable retarder with axes not parallels to \( \mathbf{e}_s \) and \( \mathbf{e}_p \), but Eq. (5) is slightly more complicated in these cases. We note \( f \) the transformation that connects \( A \) and \( A' \):

\[
f(A) = \cos \psi^* \left( A_s + \zeta A_p \right), \quad (6)
\]

with \( \zeta \), the complex number defined as

\[
\zeta = \tan \psi^* \exp (i\delta^*). \quad (7)
\]

The elimination of light occurs for \( f(A) = 0 \), that is to say for \( \zeta = -A_s/A_p \), which corresponds to an analyzer angle such as

\[
\tan \psi^* = \frac{|A_s|}{|A_p|} \quad (8)
\]

and to a retarder phase shift

\[
\delta^* = -\delta + \pi. \quad (9)
\]

When several scattering sources are present, the field in one direction is the sum of the different contributions. For instance in the case of two scattering sources, the total field can be written

\[
A = A_1 + A_2 + A_{12}, \quad (10)
\]

where \( A_1 \) and \( A_2 \) corresponds to sources 1 and 2 alone and \( A_{12} \) to their interaction. Notice that quantities present in Eq. (10) are vectorial and complex fields that comprise the phase in their complex argument (cf. Eq. (2) and Eq. (3)). Notice also that \( A_{12} \) is defined as the difference between the total scattered field \( A \) and the sum \( (A_1 + A_2) \) of the fields of the scattering sources 1 and 2 taken alone; all field being solutions of Maxwell equations in the presence of the 2 sources (case of field \( A \)) or in the presence of one or the other source (case of fields \( A_1 \) or \( A_2 \)).

If the contributions have different ellipsometric parameters, it is possible to eliminate one or several of them in order to put in evidence one of the contribution. We can, for instance, choose the parameters (Eq. (8) and Eq. (9)) such as \( f(A_1) = 0 \), the remaining signal will then be
\[ f(A_2 + A_1) \] (\( f \) being a linear transformation). Such procedure allows for instance, the reduction or the elimination of the light scattered by an interface in order to enhance the contrast of an object situated underneath. Since the ellipsometric properties of the field to eliminate must be known, the procedure generally requires the use of a scattering modeling.

3. Description of the rough surface

We consider a rough interface defined by \( z = h(x, y) \) (cf. Fig. 1 for the axes). The roughness of the surface is defined by

\[
R_q^2 = \frac{1}{S} \int_S (h(r) - \langle h \rangle)^2 \, dr,
\]

where \( S \) is the area of the considered part of the surface, \( \langle h \rangle \) the mean value of \( h \), and \( r = (x, y) \).

The correlation length of the rough surface is defined as the width at half height of the autocorrelation function \( \Gamma(r) \) of \( h(r) \),

\[
\Gamma(r) = \frac{1}{S} \int_S h(r') h(r' + r) \, dr'.
\]

We also define the slope \( w \) of the rough surface as the root mean square of the derivative of the surface height:

\[
w^2 = \frac{1}{S} \int_S |\nabla h(r)|^2 \, dr.
\]

The roughness, the correlation length and the slope are not independent. As a rule of thumb, the order of magnitude of the correlation length is \( R_q^2 / w \) [18, 19], but the exact relationship depends on the shape of the surface spectrum (Fourier transform of \( h(x, y) \)).

In order to use realistic topographies, the profiles used for the simulations, were obtained from the profile of a real rough glass surface, measured by atomic force microscopy. The roughness of this surface was 125 nm and its slope 11.7%. Profiles of different slopes and roughness were then obtained by adjustment of the horizontal and vertical scales of the initial profile.

The rough sample is considered constituted of a non-linear, non-magnetic (permeability \( \mu = \mu_0 \)) and isotropic medium of refractive index \( n \). The superstrate refractive index is considered equal to 1. All examples presented in this paper consider non absorbing media since the main interest is imaging under the rough interface, however the procedure and the theoretical expression of the parameters are also valid for absorbing media.
4. Theoretical determination of parameters $|A_p|/|A_s|$ and $\delta$

4.1. General case: differential method

The differential method allows the determination of the parameters for both rough surfaces and heterogeneous bulks. This rigorous method, which was initially designed for periodic structures [20, 21] and was later extended to non-periodic ones [22, 23], is not restricted to low roughness or long correlation lengths. However to keep reasonable calculation times (of the order of one hour on a 2 GHz computer) we restrain ourselves to surfaces with lengths of a hundred of wavelengths and with roughness of a couple of wavelengths or less, in the case of surfaces invariant along one dimension.

The differential method is able to address the general case of scattering by three dimensions structures, but is limited by numerical constraints to the study of 3D structures of the size of a few wavelengths. For this reason we will only use, in what follows, the computer code designed for structures with one dimension of invariance. However we will see that the results are in excellent accordance with those obtained by both approximate 3D theories presented below, in their respective domains of validity.

The differential method has known several improvements since its first formulation: the efficient $S$-matrix algorithm [13] for dealing with thick samples and the use of appropriate factorization rules [12, 14] for faster convergence. All these improvements are included in our computer code. We will not expose here the differential method into details since it is a rather complicated theory which doesn’t allow simple analytic expression of the scattered field. For a detailed presentation, the reader should refer to [23, 24].

4.2. Case of rough surfaces with large correlation lengths: local reflections model

In the case of important correlation length as compared to the wavelength, the diffraction process is weak and the scattering can be modeled in a physical optics approximation [17]. Scattering can then be interpreted as the result of the many specular reflections on the rough surface considered locally plane. In the frame of this model, and if we consider only light scattered in the incident plane, the scattering angle $\theta$ is linked to the incident angle $\theta_i$ and to the local angle $\theta_{\text{local}}$ (Fig. 3) by

$$\theta = 2\theta_{\text{local}} - \theta_i$$

or equivalently,

$$\theta_{\text{local}} = \frac{\theta + \theta_i}{2}.$$

The complex amplitudes, $A_p$ and $A_s$, of the field after reflection can be written as a function of the incident field amplitudes, $A_{0p}$ and $A_{0s}$, and the complex reflection coefficients, $r_p$ and $r_s$,

$$A_p = r_p A_{0p}.$$
and

\[ A_s = r_s A_{0s}, \quad (17) \]

where \( r_p \) and \( r_s \) are the Fresnel coefficients,

\[ r_p = \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2} \quad (18) \]

and

\[ r_s = \frac{\beta_1 - \beta_2}{\beta_1 + \beta_2}. \quad (19) \]

The coefficients \( \beta_1 \) and \( \beta_2 \) are defined by

\[ \beta_1 = k \cos \left( \frac{\theta_1 + \theta}{2} \right) \quad (20) \]

and

\[ \beta_2 = k \sqrt{n^2 - \sin^2 \left( \frac{\theta_1 + \theta}{2} \right)}. \quad (21) \]

The \( |A_p|/|A_s| = |r_p A_{0p}|/|r_s A_{0s}| \) ratio is then

\[ \frac{|A_p|}{|A_s|} = \frac{\beta_1 + \beta_2}{|n^2 \beta_1 - \beta_2|} \frac{|A_{0p}|}{|A_{0s}|}, \quad (22) \]

and the polarimetric phase shift \( \delta \) is given by

\[ \delta = \arg \left( \frac{r_p A_{0p}}{r_s A_{0s}} \right). \quad (23) \]

In the case of a dielectric medium, the \( r_s \) coefficient is real and positive, whatever the local reflection angle, and the \( r_p \) coefficient is real but vanishes and changes of sign when the local incidence angle, \( \theta_{\text{local}} \), goes beyond the Brewster angle, that is to say when

\[ \frac{\theta_1 + \theta}{2} = \arctan(n). \quad (24) \]

The \( r_p/r_s \) ratio is then real and positive when the local incidence angle is less than the Brewster angle, which implies \( \delta = \arg(A_{0p}/A_{0s}) \), and real and negative beyond, which implies \( \delta = \arg(A_{0p}/A_{0s}) + 180^\circ \). If we assume an incident field linearly polarized at 45° of the s direction, the scattering pattern for \( \delta \) is then a step from 0° to 180°, with a scattering angle corresponding to the step

\[ \theta_{\text{step}} = 2 \arctan(n) - \theta_i. \quad (25) \]

For instance for a rough glass sample \( (n = 1.5) \) and an incidence of 50°, the step occurs at \( \theta = 62.62^\circ \).

For an absorbing medium, \( \delta(\theta) \) follows a similar pattern but the step is replaced by a smoother transition from 0° to 180°.

4.3. Case of rough surfaces with low roughness: first order electromagnetic theory

In the case of a surface with low roughness, the scattered electromagnetic field can be considered as a perturbation of the local field and can be calculated with a first order approximation [15, 16].
In these conditions, the scattered light intensity, is proportional to the roughness spectrum \( \gamma(\sigma - \sigma_0) \),

\[
l(\sigma) = C(\sigma, \sigma_0) \gamma(\sigma - \sigma_0),
\]

with \( \gamma(\sigma) \) defined as

\[
\gamma(\sigma) = \frac{4\pi}{S} |\tilde{h}(\sigma)|^2,
\]

where \( \tilde{h}(\sigma) \) is the Fourier transform of the surface profile \( h(x, y) \) and \( S \) is the area of the light spot. \( \sigma_0 \) is the projection on the \( (Oxy) \) plane of the \( k \) vector of the incident wave and \( \sigma \) the projection on \( (Oxy) \) of the \( k \) vector of the scattered wave in the direction \( \theta \). The coefficient \( C(\sigma, \sigma_0) \) only depends on the values of the refractive index, the polarization (\( s \) or \( p \)), the wavelength \( \lambda \), and \( \sigma \) and \( \sigma_0 \). This \( C(\sigma, \sigma_0) \) coefficient does not depend on the particular microstructure of the sample. All the information concerning the microstructure is contained in the term \( \gamma(\sigma - \sigma_0) \). Let’s note that in the frame of the first order theory, the crossed polarization (\( s \rightarrow p \) or \( p \rightarrow s \)) is zero in the incidence plane.

Furthermore, it can be shown [16] that, for a dielectric rough surface lighten with a sufficiently high incidence angle \( \theta_i \) in \( p \) polarization, there is an angle \( \theta_b \) for which the \( C \) coefficient vanishes and changes of sign. This angle, called pseudo-Brewster angle, has the value

\[
\theta_b = \arcsin \left( \sqrt{\frac{n^2(n^2 - \sin^2 \theta_i)}{n^2 + (n^2 - 1) \sin^2 \theta_i}} \right).
\]

This change of sign means a step of 180° for the polarimetric phase shift, similar to the one observed with the local reflections model.

For a rough glass sample \( (n = 1.5) \) and an incidence of 50°, the \( p \) polarized scattered light vanishing and the corresponding polarimetric phase shift step, occur for a scattering angle \( \theta_b = 63.98° \). The slight difference with the value obtained by the local reflections model can be attributed to the diffraction process and is not contradictory since both methods have different domains of validity.

As for the local reflections model, the step is replaced by a smoother transition in case of an absorbing medium.

### 5. Case of surfaces with high slopes

In the case of surfaces with very low roughness to wavelength ratio \( R_q \ll \lambda \), the ellipsometric parameters, \( |A_p|/|A_s| \) and \( \delta \), vary slowly as a function of \( \theta \) and are independent of the rapid variations of the intensity which form the speckle [9]. When the roughness increases and the slope remains moderated \( (\lesssim 20\%) \), it was experimentally shown [7] that rapid variations (at the speckle scale) appear on the polarimetric phase shift. However, simulations showed that these variations remain of low amplitude as long as the slope is moderated [25]. This is no more the case for slopes greater than approximately 20%.

Calculation presented Fig. 4 shows the variation of the ellipsometric parameters, \( |A_p|/|A_s| \) and \( \delta \), as a function of the scattering angle, in the case of a rough dielectric sample with a high slope \( (\omega = 100\%, \ i.e. \ 45°) \). Calculation were performed with the differential method and results given by the approximate first order theory are also presented Fig. 4. It can be observed that the parameters vary with a large amplitude at the speckle scale, even though the roughness is moderated \( (R_q = 100 \text{ nm}) \). The elimination of such polarized field requires the adjustment of the analyzer angle and phase retarder for every speckle grain, which could more easily be done using specific devices, such as pixelized liquid crystal matrices for instance.

Contrary to low scattering level, the ellipsometric parameters here depend on the microstructure of the sample and not only of its statistical properties. The selective elimination procedure
should then first require the measurement of the surface topography, followed by the calculation of the ellipsometric parameters by a rigorous three dimensional method. However, most rigorous three dimensional methods are restricted by numerical constraints to the study of the scattered light corresponding to a light spot of a few microns in diameter, which is far from the usual experimental conditions. Notice that it is still possible to adjust experimentally the elimination parameters in order to cancel or minimize the total scattered field, similarly to nulling ellipsometry methods [26].

6. Case of surfaces with moderated slopes

The case of surfaces with moderated slope ($\leq 20\%$) is interesting since it constitutes a vast majority of samples. The other reason that makes this case interesting is, as we will see, the fact that the light scattered by this type of surface can be eliminated efficiently without any knowledge of the particular microstructure and therefore without the necessity of a mounting with an angular resolution as high as the speckle one.

6.1. Difference between a low slope surface and a high slope surface for a same roughness value

Figure 5 represents the phase difference $\delta$ and the $|A_p|/|A_s|$ ratio in the same conditions as for Fig. 4, but for a low slope: $w = 2\%$ (or $1.15^\circ$). Even though the roughness is the same, $R_q = 100$ nm, the oscillations are small compared to the case of the high slope surface. The results presented here, not only present few oscillations, but are also in good accordance with those of first order theory.

The fact that the polarization direction and its ellipticity (or equivalently $|A_p|/|A_s|$ and $\delta$) vary little at the speckle scale, doesn’t mean that it is the same for the intensity. If we take, for instance, a rough surface with a slope of $15\%$ and a roughness of $500$ nm, we observe (Fig. 6) that the intensity, in polarization $s$ and $p$, vary with a large amplitude ($\sim 1$ decade) at the speckle scale. On the other hand, when we observe the details of these variations, we see that the oscillations are strongly correlated between $s$ and $p$. This behavior explains that the $|A_p|/|A_s|$ ratio show slight variations from one speckle grain to another.
Fig. 5. Ellipsometric parameters $|A_p|/|A_s|$ and $\delta$ of the field scattered by a rough surface with low slope ($w = 2\%$, i.e. $1.15^\circ$) and a roughness of 100 nm. Same conditions and medium properties as for Fig. 4.

Fig. 6. $s$ and $p$ intensity scattered by a rough surface with moderated slope ($w = 15\%$) and a roughness of 500 nm. Same conditions and medium properties as for Fig. 4.

Our simulations show that it is the same for the phase difference: the $\delta_p$ and $\delta_s$ phases show large variations but in a correlated manner that makes their difference close to zero or $180^\circ$ (Fig. 7).

One important point to keep in mind is that the ellipsometric parameters, $\delta$ and $|A_p|/|A_s|$, of the scattered field, vary slowly with the scattering angle when the slope of the surface is moderated (cf. Fig 5). This point constitutes an important difference with the case of high slope surfaces for which the rapid variations of $s$ and $p$ amplitudes and phases are not correlated and lead to large variations of the ellipsometric parameters $|A_p|/|A_s|$ and $\delta$ at the speckle scale (cf. Fig. 4).

6.2. Roughness influence in the case of a moderated slope

Figure 8 shows the evolution of the $|A_p|/|A_s|$ ratio, simulated by the differential method and plotted as a function of $\theta$, for several surfaces with the same slope $w = 15\%$ and for increasing roughness. The results obtained by the local reflections model and by the first order theory are also presented Fig. 8.

Roughness varies from $R_q = 1$ nm, which corresponds to a super-polished glass sample, to
$R_q = 500$ nm, a roughness of the same order of the wavelength, for which all the incident light is scattered (no specular beam remaining).

When the roughness is low ($R_q = 1$ nm) the results are in perfect accordance with those of first order theory. When the roughness increases ($R_q = 20$ nm), rapid oscillations appear in the $|A_p|/|A_s|$ pattern, but remain of limited amplitude. When the roughness is still increased ($R_q$ from 50 to 500 nm) we observe that the amplitude of the oscillations doesn’t increase significantly and that the average position of the curve moves slowly from the one corresponding to the first order theory to the one corresponding to the local reflections model.

Let’s first notice that the three theories used for the determination of the scattered field are in good accordance: the results obtained by the differential method tend towards those of the first order theory when the roughness is low and to those of the local reflections model when the roughness is high (for a fixed slope, high roughness correspond also to large correlation lengths). Due to numerical constraints, we consider the surface to be invariant along one dimension when using the differential method. We observe that, even with this approximation, the results are in accordance with those obtained by both approximate theories which are 3D theories.

Another interesting result is the fact that the oscillations remain of limited amplitude whatever the roughness value. When the roughness increases, the first order theory, which predicts no oscillations, is no longer valid, but since the correlation length increases, the domain of validity of the local reflection model, which also predicts no oscillations, becomes closer. This explains that the oscillations stay bounded when the slope is moderated.

Let’s note that in order to maintain acceptable calculation times (a few hours), all profile were limited to lengths of 50-100 μm, which is less than usual spot light diameters. It is possible that the profile length has an influence on the oscillations amplitude, but the few calculations we made to answer this question doesn’t show a clear tendency and suggest a weak influence of the length.

The polarimetric phase difference $\delta$ follows the same behavior than $|A_p|/|A_s|$: oscillations around the mean value remain of limited amplitude as long as the slope is moderated ($\lesssim 20\%$), cf. [25].

The results of Fig. 8 show that the selective reduction of light scattered by a rough interface with a moderated slope, should not require a setup allowing the adjustment of the parameters.
Fig. 8. $|A_p|/|A_s|$ for several roughness, in the case of a rough surface with a 15% slope. Normal incidence, wavelength of 632.8 nm, refractive index equal to 1.5.
for each speckle grain and, more importantly, that the particular sample topography wouldn’t have to be taken into account in the elimination procedure.

7. Application to surface scattering reduction: simulations

We want here to simulate the scattering reduction procedure by using a rigorous scattering theory (the differential method) to calculate the field scattered by the sample, in conjunction with elimination parameters, obtained by an approximate theory. Even though the differential method could also be used to calculate the scattering reduction parameters (which should, in principle, lead to a more efficient reduction since the method is rigorous) this would not be practical since the parameters would depend on the particular microstructure of the sample, which is not the case for the approximate theories presented in this paper.

Figure 9 presents a simulation of reduction of the light scattered by a rough interface with moderated slope. The intensity of the $s$ and $p$ components of the field scattered by the same rough surface is represented Fig. 6. The slope of the surface is 15% and the roughness 500 nm. The incident field is linearly polarized at 45° of the $s$ direction and has an incidence of 50°. The scattering reduction parameters, analyzer angle $\psi^*$ and phase delay $\delta^*$, are those obtained by the local reflection model and are represented Fig. 9. The intensities before and after reduction of the scattered light are represented in logarithmic scale. The intensity after passing through
Fig. 10. Cylindrical inclusions with refractive index 2.0, situated in a medium with refractive index 1.5, under a rough interface. Total height: 10 μm, total length: 55.8 μm.

Fig. 11. (a): Intensity scattered by cylindrical inclusions of Fig. 10, without rough interface, before and after reduction of the scattered light using the parameters of Fig. 9. The attenuation coefficient of the whole scattered light is 0.12 (compared to 0.00029 for surface scattered light reduction presented Fig. 9). (b): Intensity scattered by the cylindrical inclusions and the rough interface of Fig. 10, before and after reduction of the light scattered by the interface. The remaining light is comparable to the remaining light in the absence of rough interface (cf. (a)).

\[
I_{\text{annul}} \propto \cos^2 \psi^* (A_2^2 + \tan^2 \psi^* A_p^2 + 2|A_1A_p| \tan \psi^* \cos (\delta^* + \delta^*))
\]  

(29)

We observe a strong scattering attenuation after the elimination device: the ratio between total scattered intensity (integrated between $-90^\circ$ and $90^\circ$) after and before applying the reduction of the scattered light procedure is $2.9 \cdot 10^{-4}$. This result is very satisfactory since it shows the possibility of a high attenuation factor without any knowledge of the particular topography of the rough surface, as long as its slope is moderated.

Consider now the situation where the light scattered by a rough surface must be separated from the light scattered by an object situated under this surface. For the selective procedure to work it is necessary that the ellipsometric parameters of the object be clearly different from those of the rough surface; otherwise the elimination procedure will also cancel the light scattered by the object. In other words, if $A = A_1 + A_2 + A_{12}$ denotes the total field with 1 and 2 corresponding respectively to the rough surface and the object situated underneath, the relation $f(\zeta_1, A_1) = 0$ must not imply $f(\zeta_1, A_2) + f(\zeta_1, A_{12}) = 0$.

Results are given in Fig. 11 for an object constituted of cylindrical inclusions situated under a rough interface (Fig. 10). Reference results are first given in Fig. 11(a) for the object under a perfectly flat surface (field $A_2$). When the surface scattering reduction parameters are used to reduce scattering from the rough surface, that is $f(\zeta_1, A_1) \approx 0$, we observe a reduction of the reference object by a factor 0.12 which shows that $f(\zeta_1, A_2)$ is far from zero (in comparison with the 0.00029 factor obtained with the rough surface). Such successful result is connected
with the different polarization behaviors for the object ($A_2$ field) and the rough surface ($A_1$ field).

Figure 11(b) is calculated for the scattering pattern in the situation where both the rough surface and the inclusions are present. The initial scattered field is denoted $A = A_1 + A_2 + A_{12}$ and is turned after reduction of the surface scattering ($f(\zeta_1, A_1) \approx 0$) into $f(\zeta_1, A) \approx f(\zeta_1, A_2) + f(\zeta_1, A_{12})$. Notice that the angular dependence of Fig. 11(a) is not exactly recovered with Fig. 11(b). Indeed, after reduction of the light scattered by the rough interface, most of the light comes from the scattering of the cylindrical inclusion, however its angular dependence is modified by the interaction with the rough interface. Mathematically it corresponds to the term $A_{12}$ and physically to the fact that the light coming from the inclusions is scattered in transmission by the rough interface.

8. Application to surface scattering reduction: experimental demonstration

In order to test the predictions and obtain a first experimental validation, we worked with a fused silica sample of thickness 3.5 mm, diameter 25 mm and refractive index 1.46. The top surface of the sample has a roughness of 7.2 $\mu$m and a correlation length of 32 $\mu$m (measured by a white light interferometer). With this roughness, the light is completely scattered (no specular reflection). The back surface of the sample is superpolished with negligible direct contribution to the scattering process.

The first step was to demonstrate the reduction of the light scattered from the rough surface. The sample was lighten by HeNe laser light ($\lambda = 632.8$ nm) linearly polarized at 45° of $s$ direction and with an angle of incidence $\theta_i = 10^\circ$. The measurements were performed with a CCD camera in the scattering direction $\theta = 15^\circ$ in the incidence plane (i.e. 25° between the direction of incidence and the direction of measurement). The applied scattering reduction parameters, determined by the local reflection model, were $\psi^* = 133.1^\circ$ and $\delta^* = 0.0^\circ$. Results are given in Fig. 12. The left and right figures were recorded with the same exposure time, before and after applying the procedure. An attenuation filter was used for the left figure in order to avoid overexposure. Comparison of these two figures highlights a reduction factor close to $10^{-3}$, which confirm the efficiency of the procedure. Notice that the light reflected by the back surface, after scattering in transmission by the front rough surface, is also reduced, since in these conditions, its parameters are close to those of the light scattered in reflection by

Fig. 12. Scattering from a highly rough surface, before (a) and after (b) application of the light scattering reduction parameters. The attenuation coefficient is around $10^{-3}$. The aperture and exposure time of the camera are identical for both images, but the intensity of the incident beam was reduced by a factor $10^2$ for image (a) to avoid overexposure.
The second step was to validate the selective property of the procedure. To reach this goal, we stuck a white paper cross with index matching gel on the back face of the sample. Results are given Fig. 13. On the left figure, the signal is dominated by scattering from the rough surface, so that the cross cannot really be distinguished. Notice that since the object is impregnated with index matching gel it scatters less than a dry piece of paper. Notice also that a reduction of the laser intensity by a factor 10 was applied for the left figure, to avoid overexposure. On the right figure, the surface scattering reduction parameters are applied; the light scattered by the surface is strongly reduced, which makes the cross clearly appear. Such contrast enhancement confirm the theoretical prediction of the previous section and validates the selective nature of the procedure.

9. Conclusion

This paper was devoted to the generalization of a first-order scattering reduction procedure to the case of arbitrary complex media. Some key difficulties had to be solved such as dependence of ellipsometric parameters versus microstructure, rapid variations of these parameters with scattering angle, and computation time of the scattering reduction parameters. Relying on an exact electromagnetic theory, We have shown that for surfaces of arbitrary roughness but moderate slopes (≲ 20%), which constitute the vast majority of samples, the ellipsometric parameters could be nearly reached with approximate theories that limit the influence of microstructure and variation with scattering direction. Such approximate parameters were then applied to enhance the contrast between a rough surface and inclusions beneath this surface, and the result was an improvement of the order of 2 decades. First experiments were performed with success to confirm these predictions, hence opening a door to the generalization of the selective imaging procedure.