

# *Gratings: Theory and Numeric Applications*

ed. E. Popov (Institut Fresnel, CNRS, AMU, 2012)

ISBN: 2-85399-860-4

## **B. Stout, Chapter 6: Spherical harmonic Lattice Sums for Gratings**

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