Sub-λ metallic surfaces : a microscopic analysis

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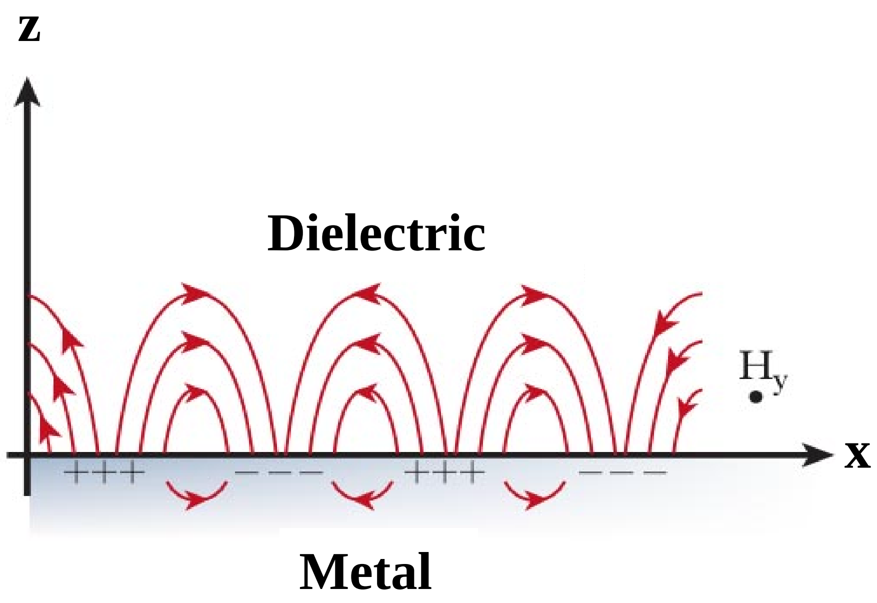
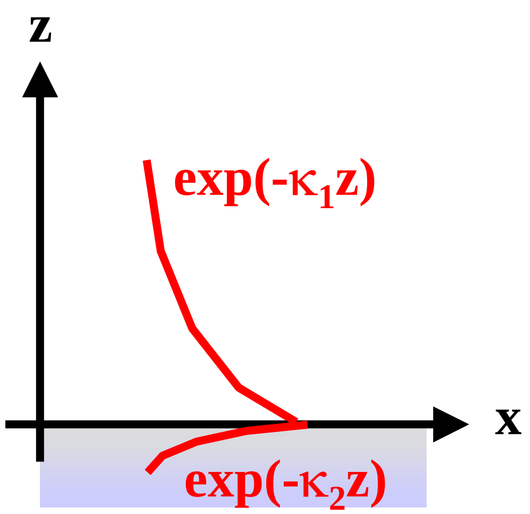
Jean-Paul Hugonin

Haitao Liu (Nankai Univ.)

Surface plasmon polariton

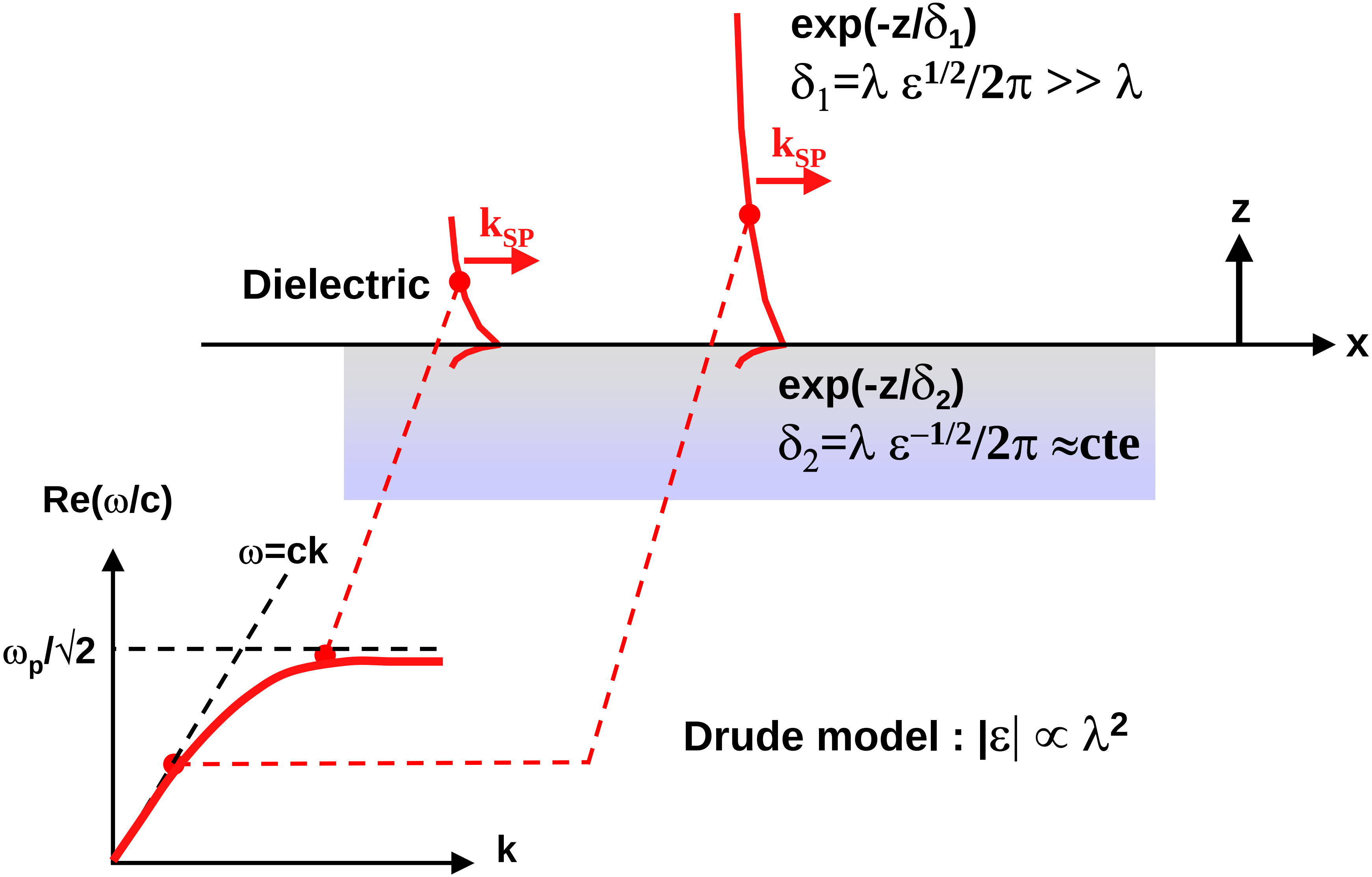




SPPs are localized electromagnetic modes/ charge density oscillations at the interfaces, which exponentially decay on both sides

$$k_{SP} = \frac{\omega}{c}\left(\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}\right)^{1/2}$$

Surface plasmon polariton



1.The emblematic example of the EOT

-extraordinary optical transmission (EOT)
-limitation of classical "macroscopic" grating theories
-a microscopic pure-SPP model of the EOT

2.SPP generation by 1D sub-λ indentation

-rigorous calculation (orthogonality relationship)
-slit example
-scaling law with the wavelength

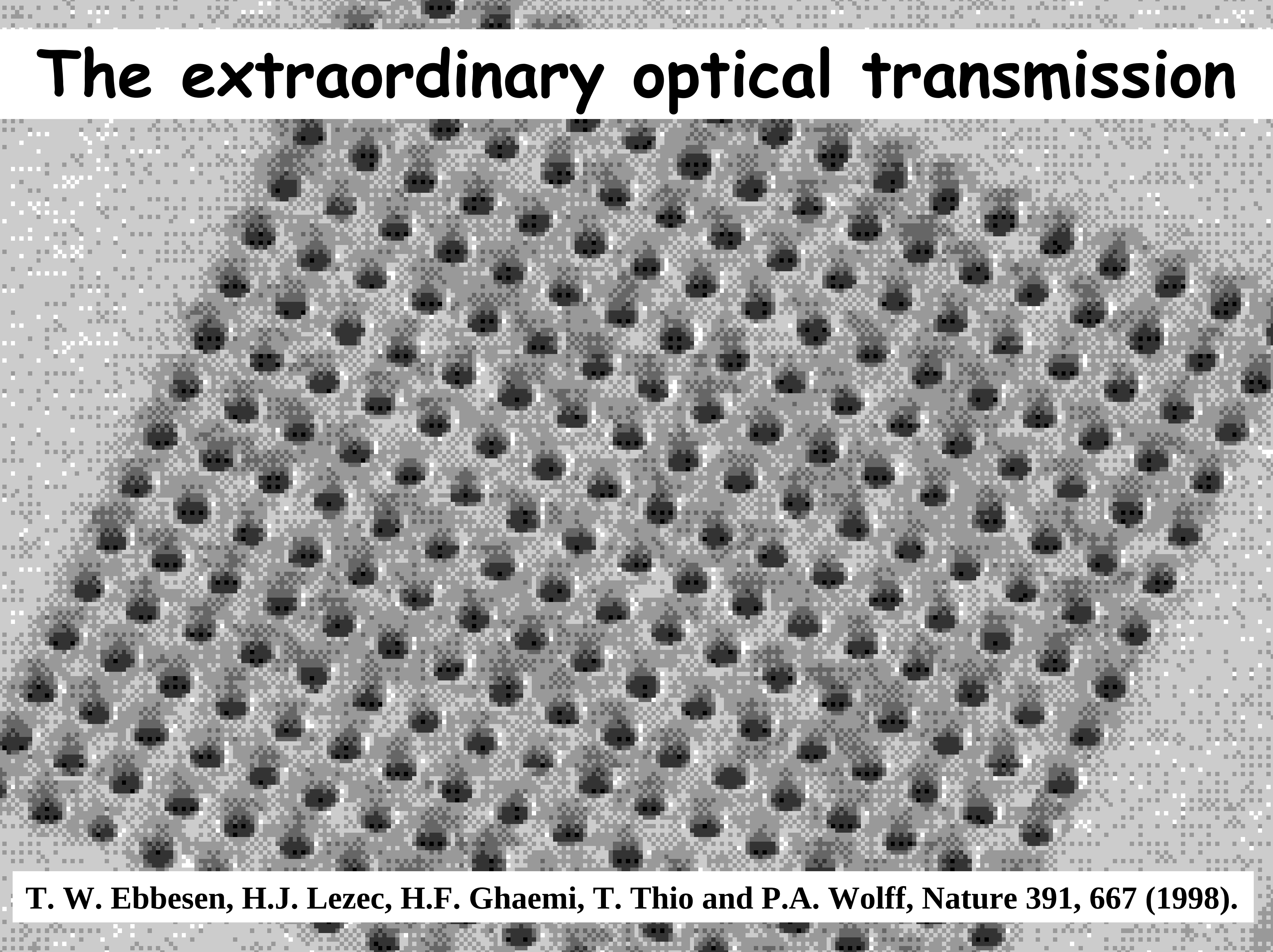
3.The quasi-cylindrical wave

-definition & properties
-scaling law with the wavelength

4.Multiple Scattering of SPPs & quasi-CWs

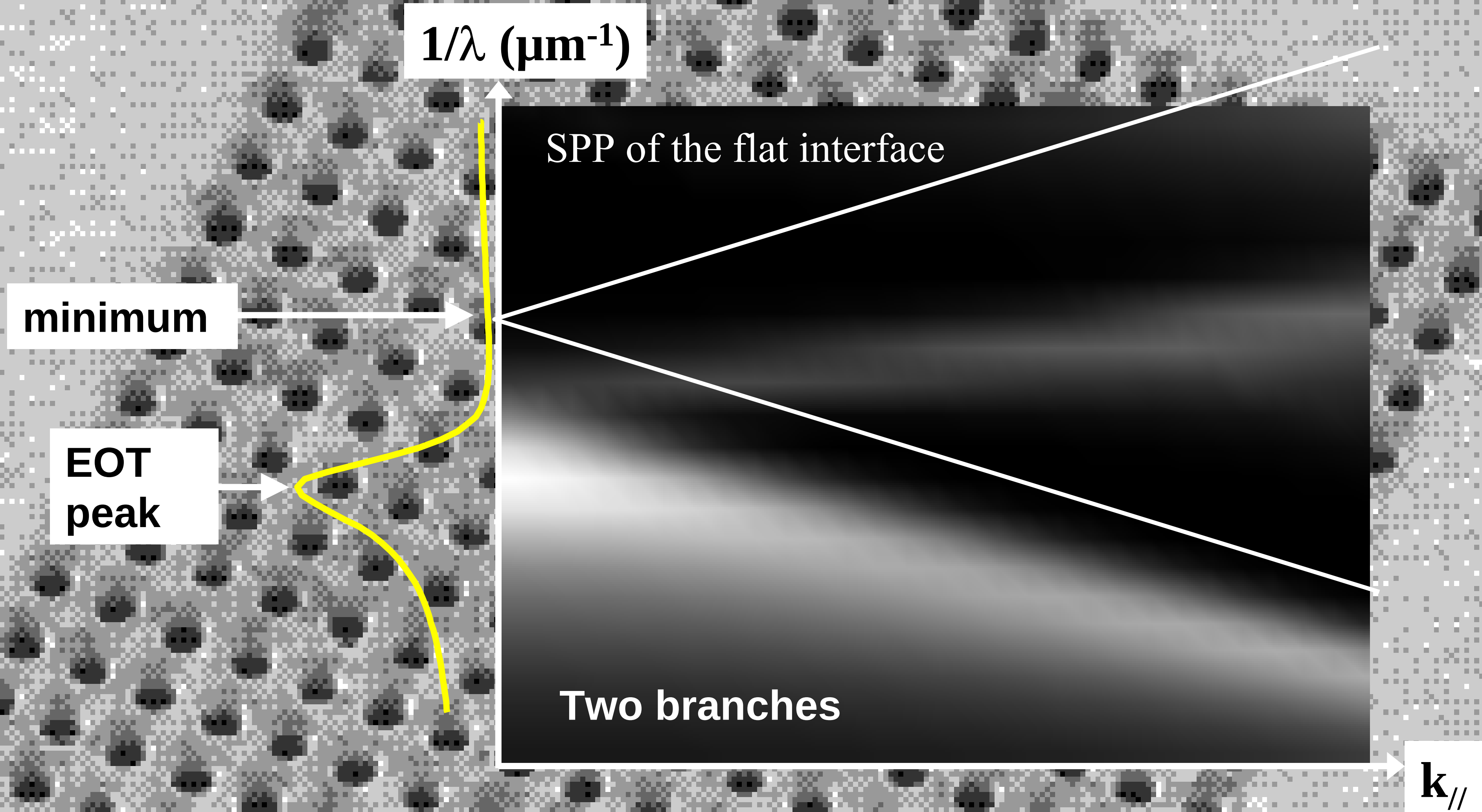
-definition of scattering coefficients for the quasi-CW

The extraordinary optical transmission

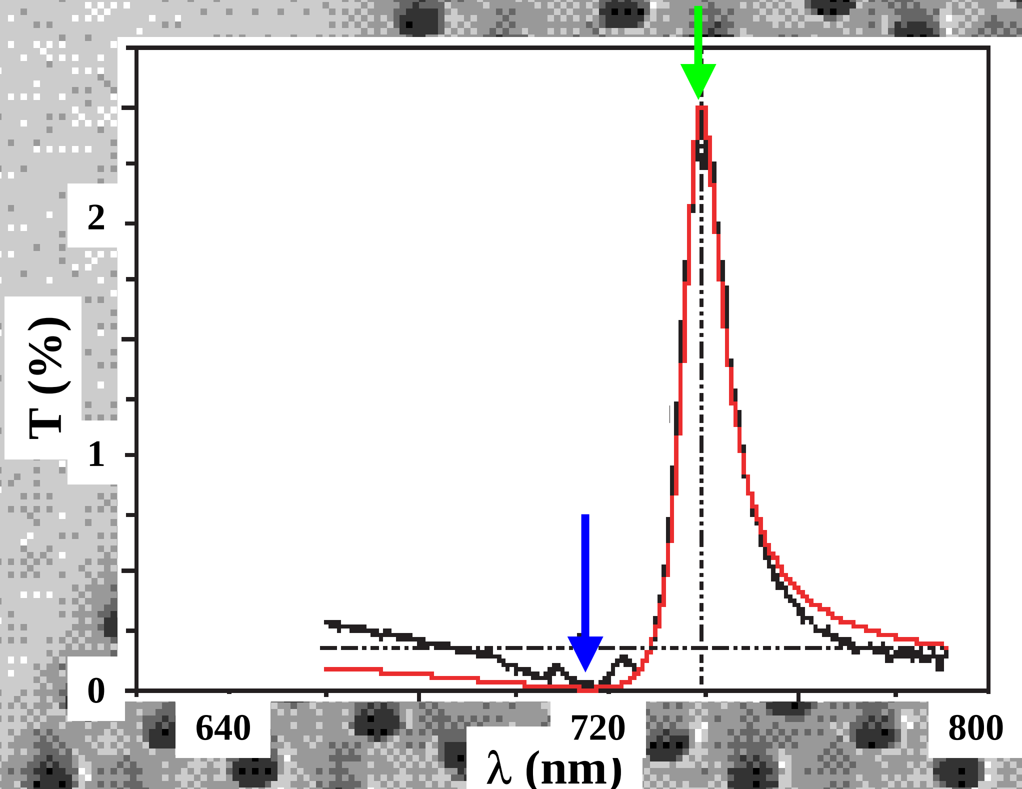


T. W. Ebbesen, H.J. Lezec, H.F. Ghaemi, T. Thio and P.A. Wolff, Nature 391, 667 (1998).

transmittance (ω,k//)



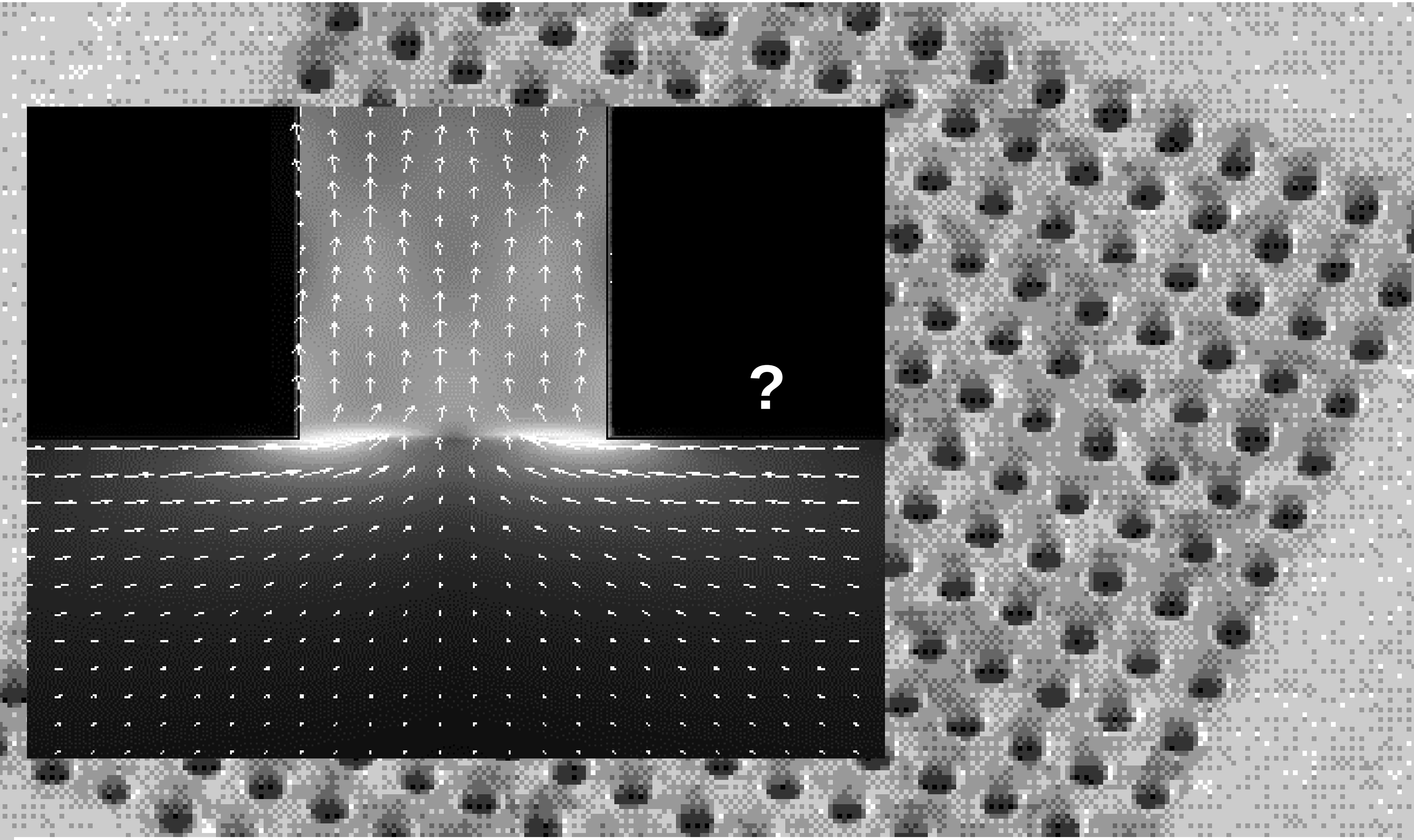
The extraordinary optical transmission



Black : experiment **red : Fano fit**

C. Genet et al. Opt. Comm. 225, 331 (2003)

The extraordinary optical transmission



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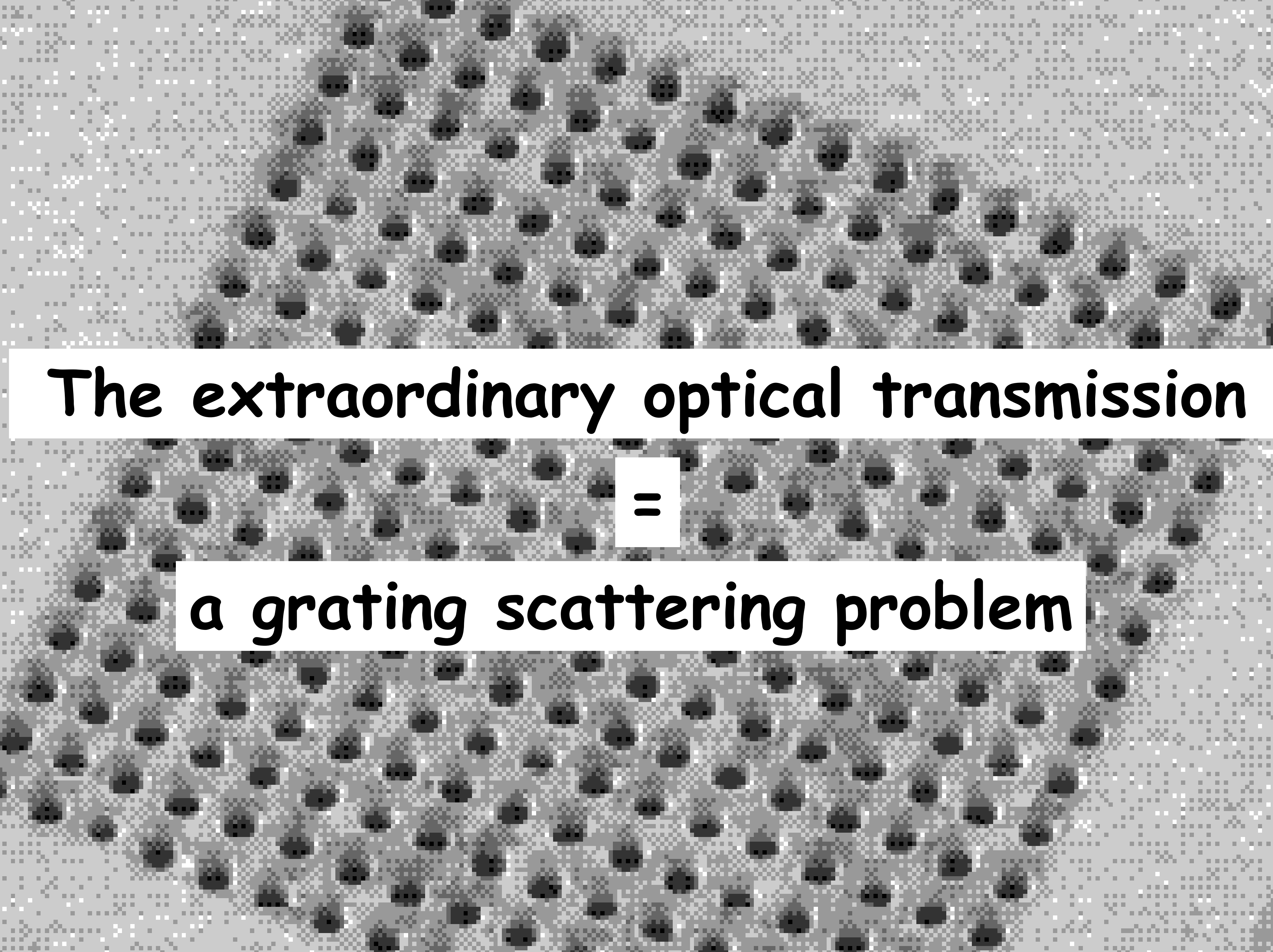

The extraordinary optical transmission
=
a grating scattering problem

What is learnt from grating theory?

XLII. *On a Remarkable Case of Uneven Distribution of Light in a Diffraction Grating Spectrum.* *By* R. W. Wood, *Professor of Experimental Physics, Johns Hopkins University**.

Phil. Mag. ***4****, 396-402 (1902).*

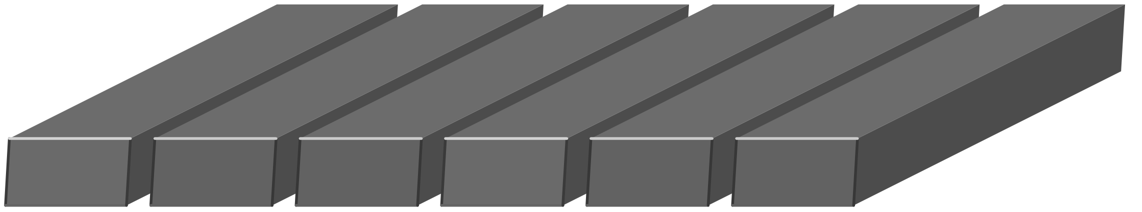
Electromagnetic Theory of Gratings

Edited by R. Petit

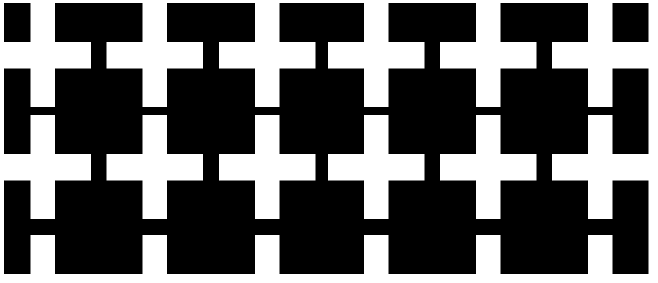
With Contributions by
L. C. Botten M. Cadilhac G. H. Derrick
D. Maystre R. C. McPhedran M. Nevière
R. Petit P. Vincent

Springer-Verlag Berlin Heidelberg New York 1980

Wire grid polarizer



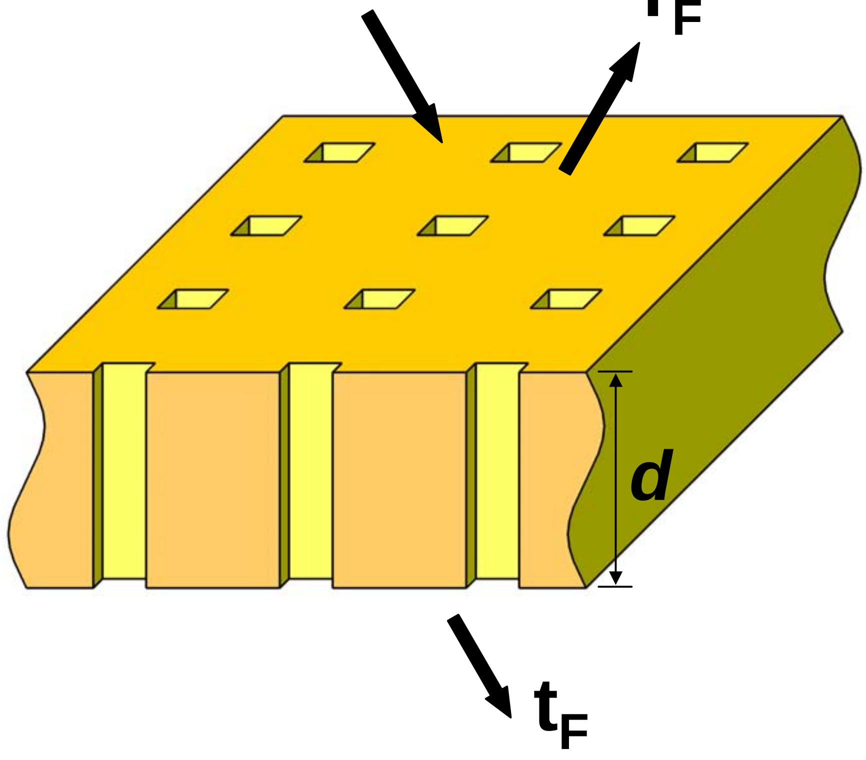
Inductive-capacitive grids



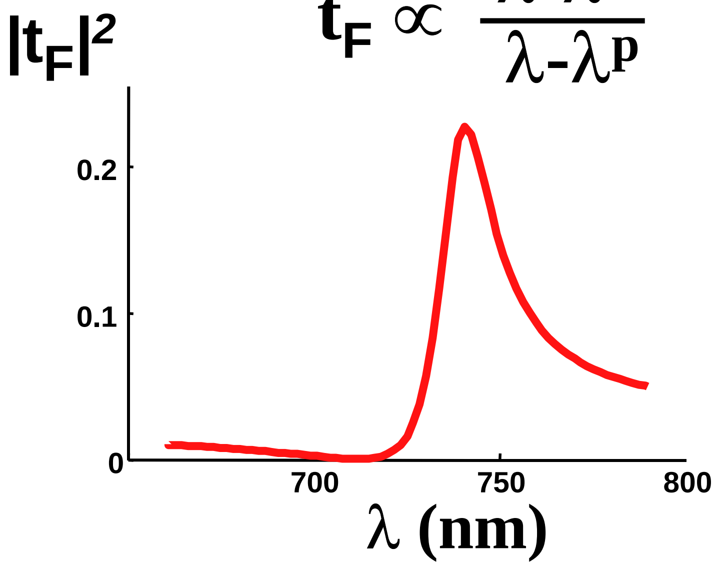
Nearly 100% of the incident energy is transmitted at resonance frequencies for TM polarization

- **Hertz (1888) first used a wire grid polarizer for testing the newly discovered radio wave.**
- **J.T. Adams and L.C. Botten J. Opt. (Paris) 10, 109–17 (1979).**
- **R. Ulrich, K. F. Renk, and L. Genzel, IEEE Trans. Microwave Theory Tech. 11, 363 (1963).**
- **C. Compton, R. D. McPhedran, G. H. Derrick, and L. C. Botten, Infrared Phys. 23, 239 (1983).**

Poles and zeros of the scattering matrix



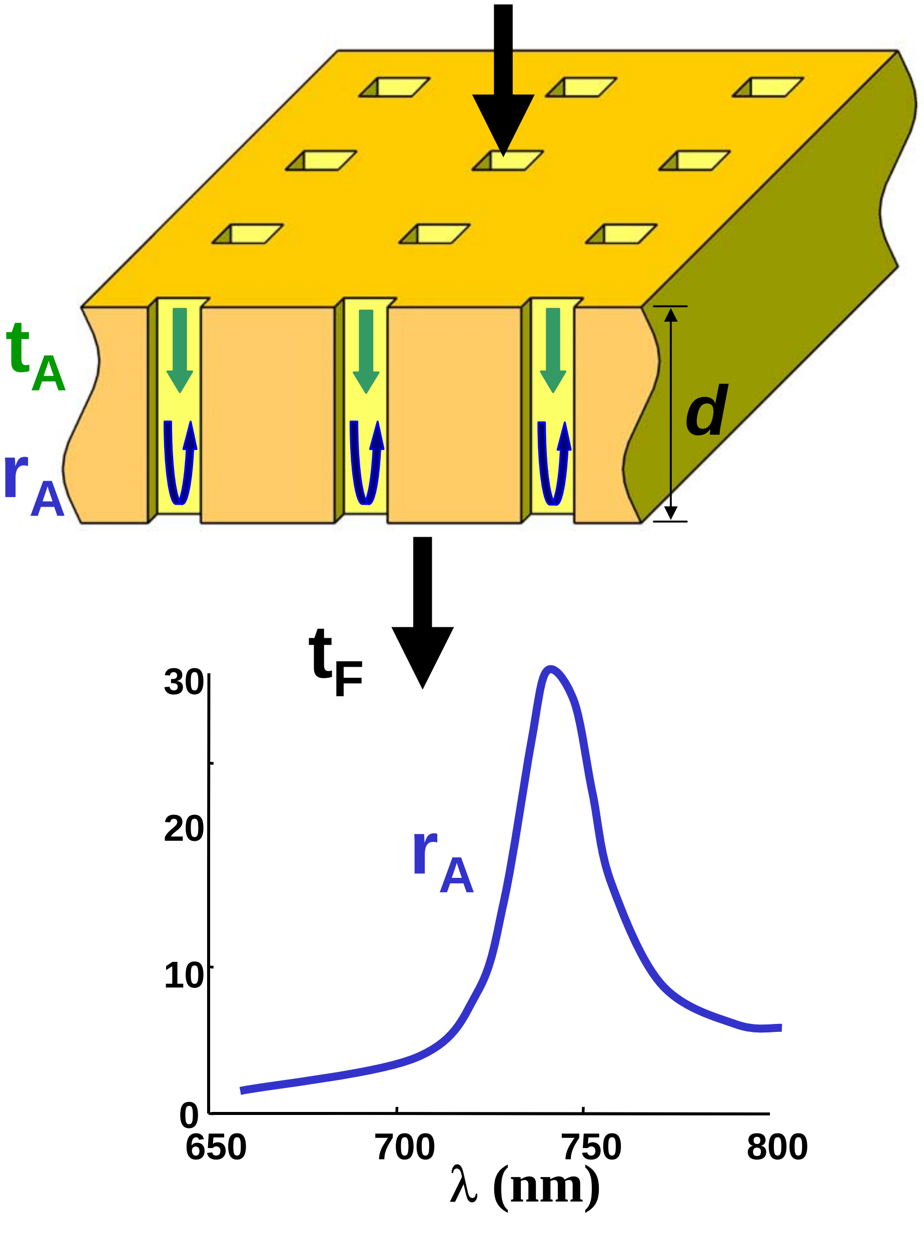
$$t_F \propto \frac{\lambda-\lambda^z}{\lambda-\lambda^p}$$



-Global analysis.
-Why does the pole exist?
Why does the zero exist?
Why are they close or not to the real axis?

E. Popov et al., PRB 62, 16100 (2000).

The surface-mode interpretation

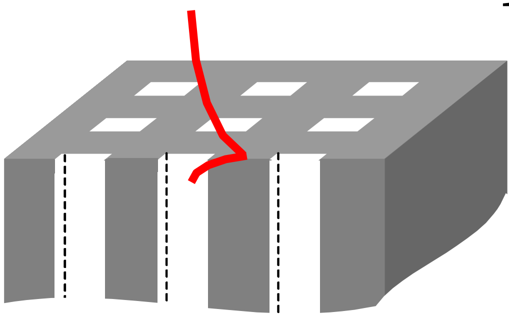


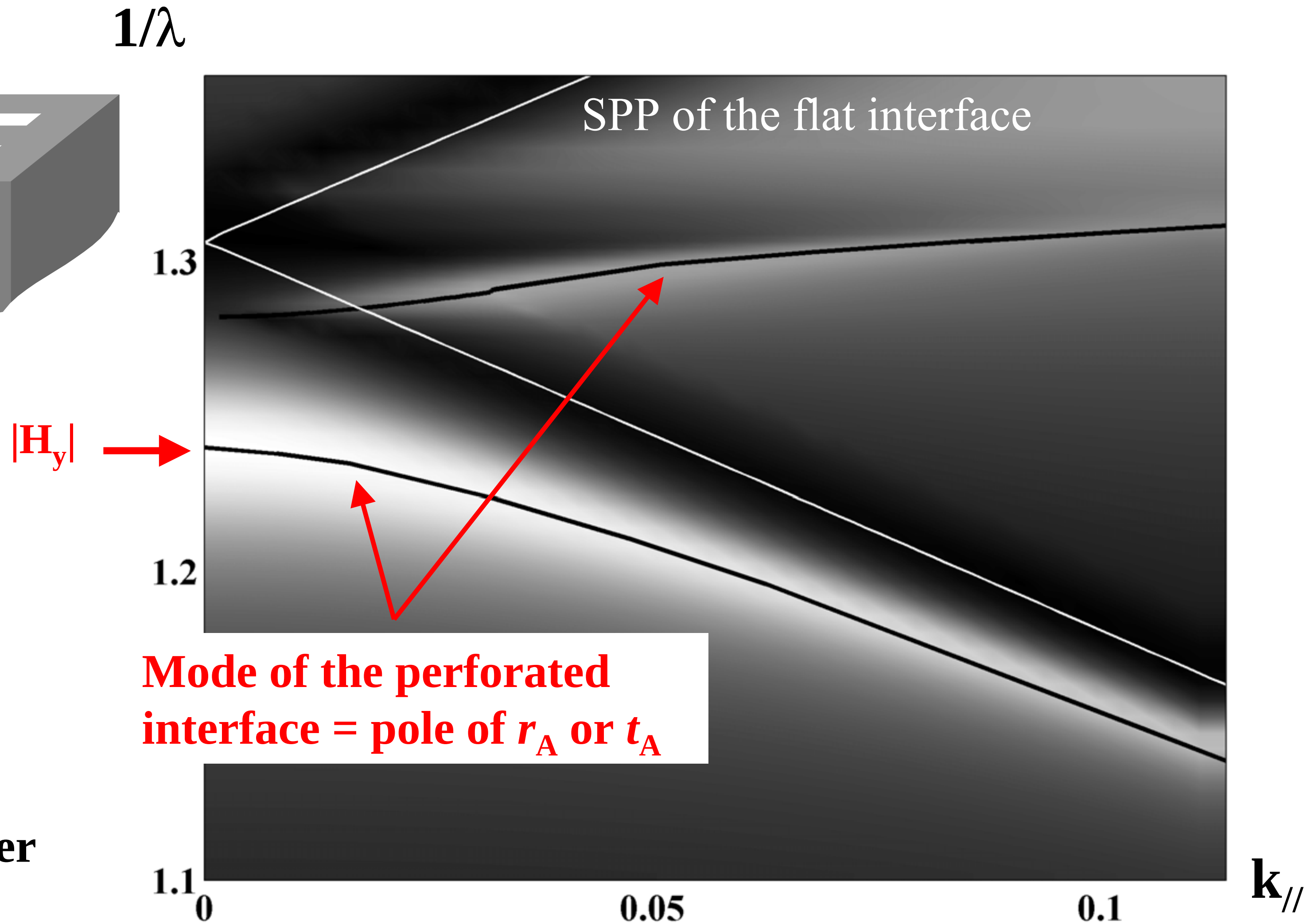
Resonance-assisted tunneling

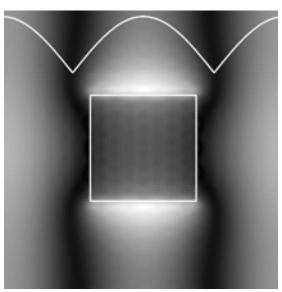
$$t_F = \frac{t_A^2 \exp(ik_0 nd)}{1 - r_A^2 \exp(2ik_0 nd)}$$

L. Martín-Moreno, F. Garcia-Vidal & J. Pendry, Phys. Rev. Lett. 86, 1114 (2001).

The surface-mode interpretation



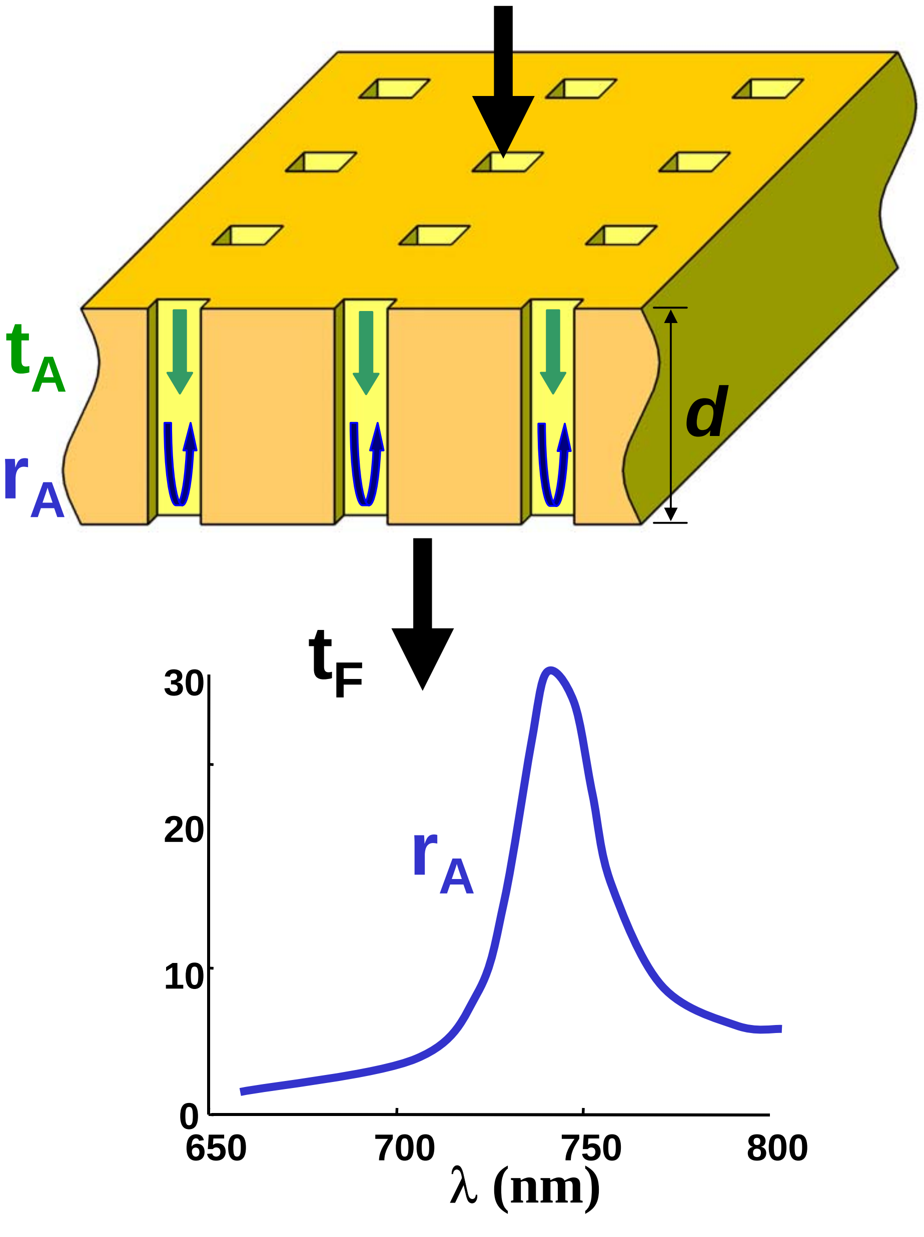




Hybrid character

P. Lalanne, J.C. Rodier and J.P. Hugonin , J. Opt. A 7, 422 (2005).

The surface-mode interpretation



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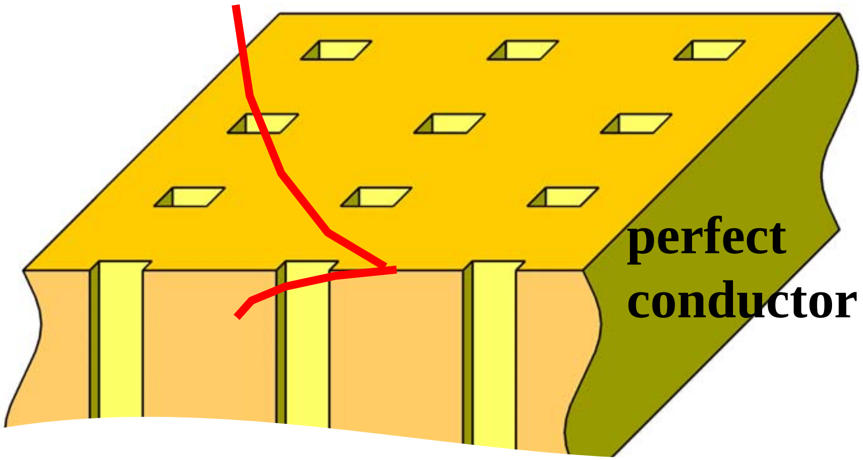
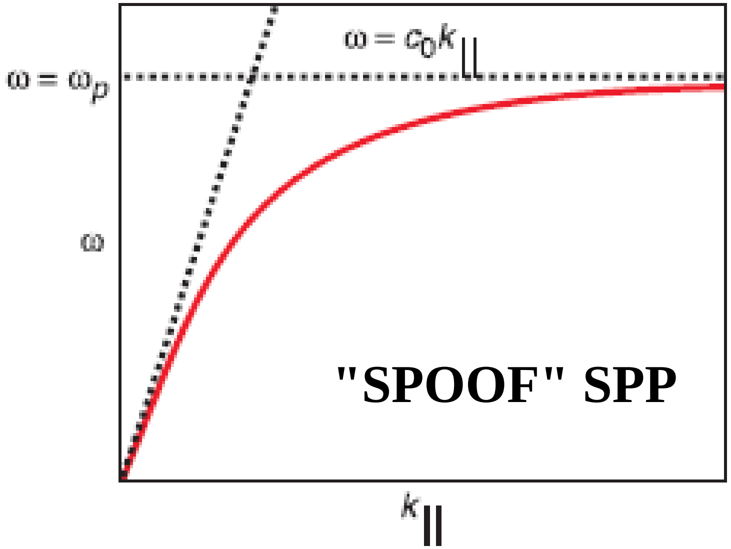
→ reinforcement of the initial vision of a SPP assisted effect

The surface-mode also exists as $\lambda \to \infty$

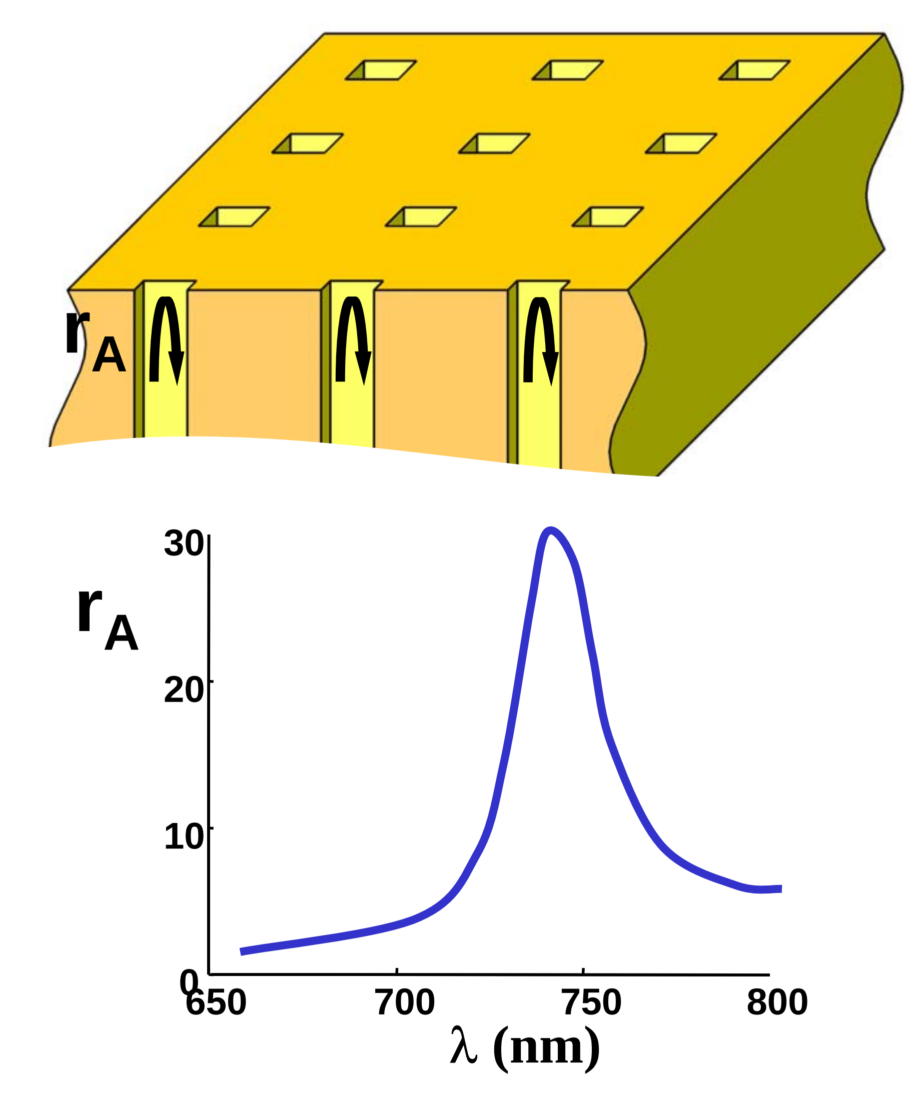




Theory :
J. Pendry, L. Martín-Moreno & F. Garcia-Vidal, Science 305, 847 (2004).

Experimental verification :
P. Hibbins et al., Science 308, 670 (2004).

Weaknesses of classical grating theories



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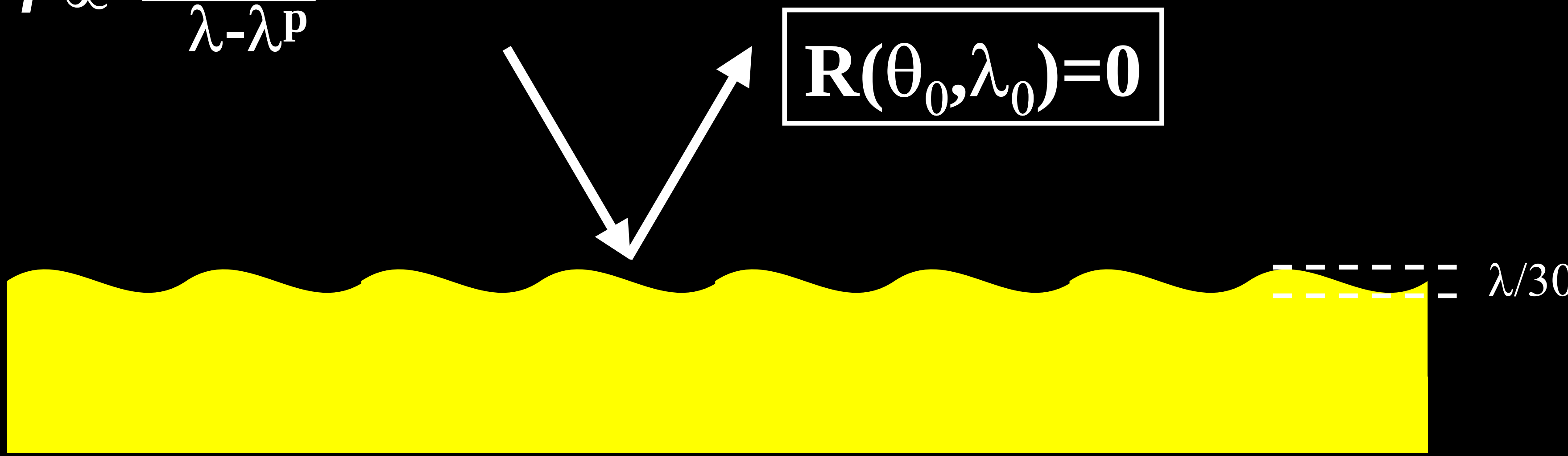
L. Martín-Moreno, F. Garcia-Vidal & J. Pendry, Phys. Rev. Lett. 86, 1114 (2001).

-The resonance of t_F is explained by the resonance of another scattering coefficients.

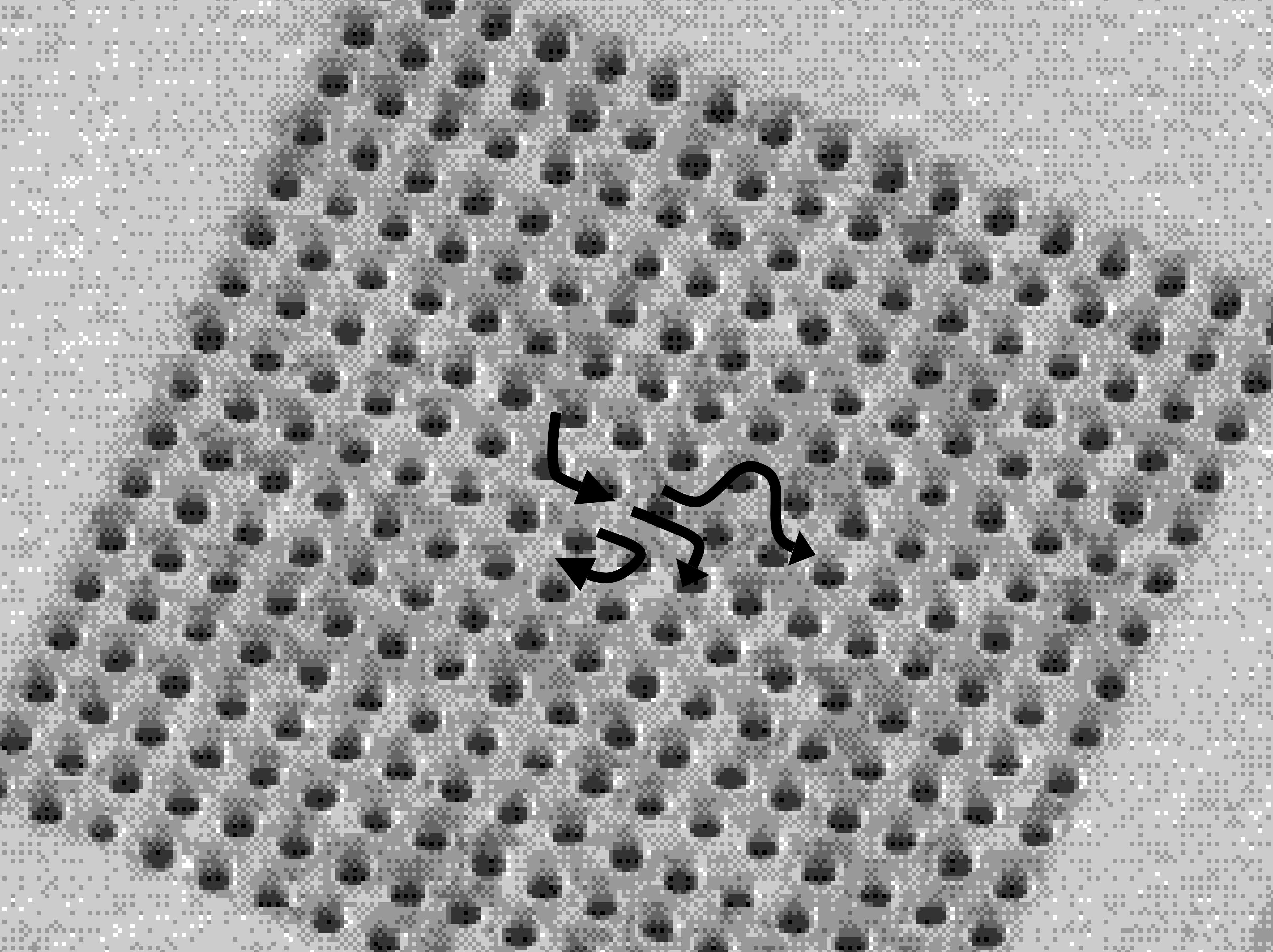
-In reality, nothing is known about the waves that are launched in between the hole and that are responsible for the EOT

$$r \propto \frac{\lambda-\lambda^{z}}{\lambda-\lambda^{p}}$$

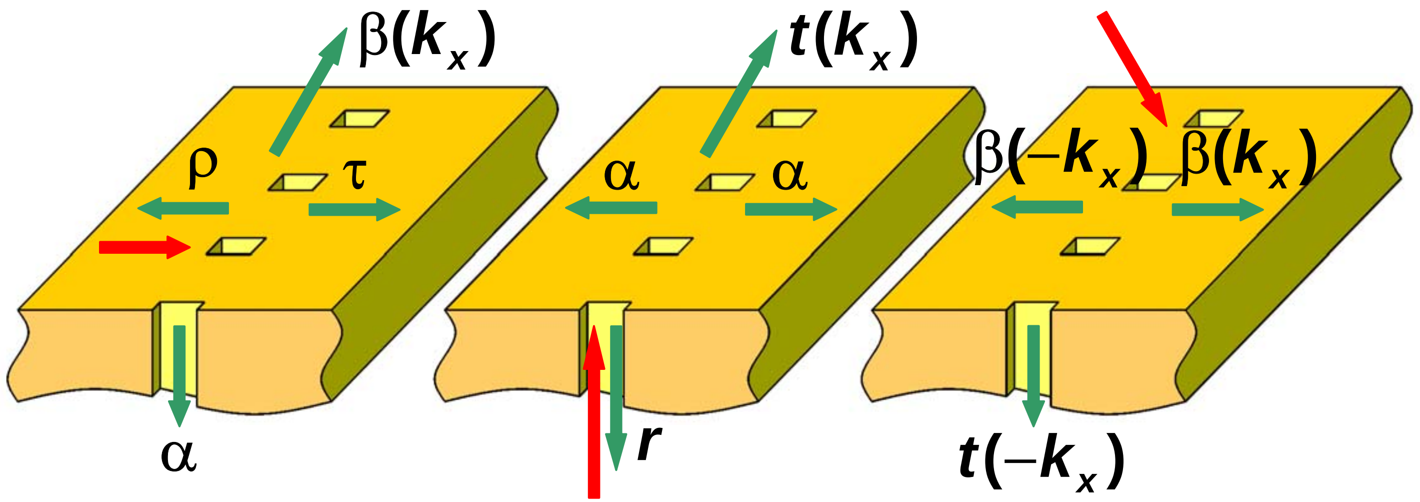


M. C. Hutley and D. Maystre, “Total absorption of light by a diffraction grating,” Opt. Commun. **19**, 431-436 (1976).

D. Maystre, General study of grating anomalies from electromagnetic surface modes, in: A.D. Boardman (Ed.), Electromagnetic Surface Modes, Wiley, NY, 1982, (chapter 17).

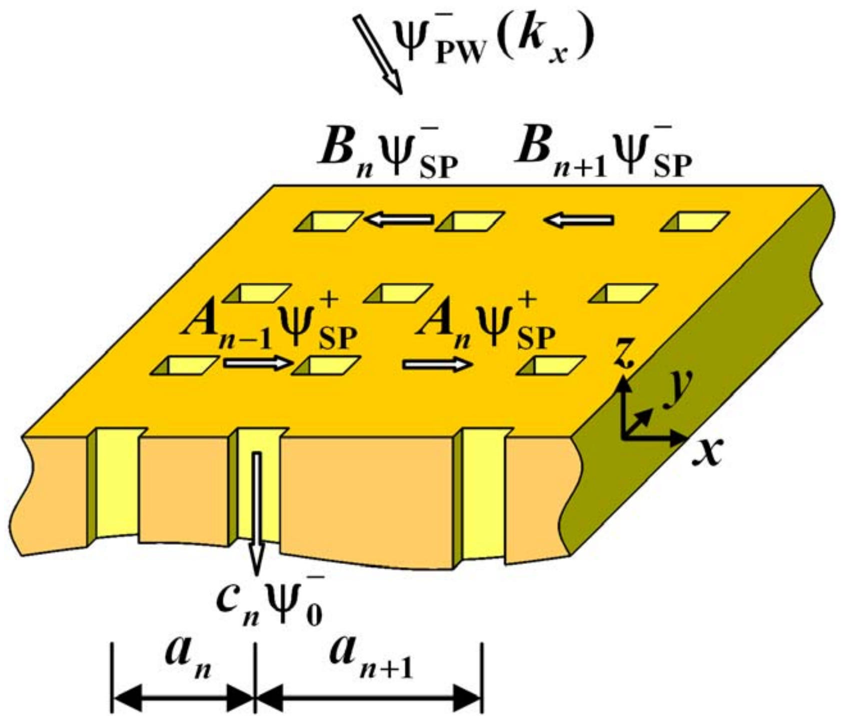


Microscopic pure-SPP model



H. Liu, P. Lalanne, Nature 452, 448 (2008).

SPP coupled-mode equations

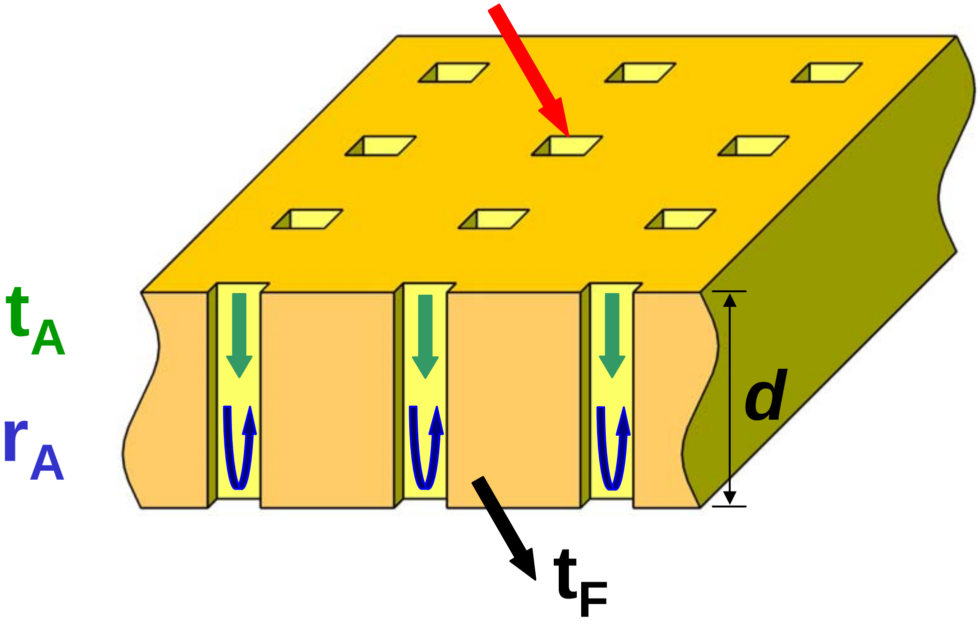


- $A_n = w_1 w_2 \ldots w_n\, \beta(k_x) + u_n \tau A_{n-1} + u_n \rho B_{n+1}$
- $B_n = w_1 w_2 \ldots w_n\, \beta(-k_x) + u_{n+1} \tau B_{n-1} + u_n \rho A_{n-1}$
- $c_n = w_1 w_2 \ldots w_n\, t(-k_x) + u_n \alpha A_{n-1} + u_{n+1} \alpha B_{n+1}$

Periodicity is not needed!

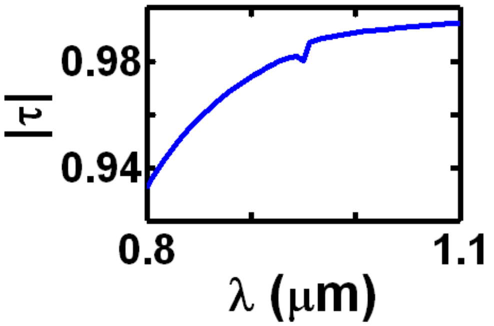
with $u_n = \exp(ik_{SP}a_n)$, $w_n = \exp(ik_x a_n)$

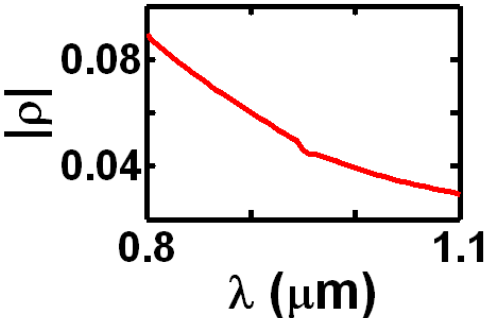
Microscopic pure-SPP model



$$t_F = \frac{t_A^2 \exp(ik_0nd)}{1- r_A^2 \exp(2ik_0nd)}$$

$$t_A = t + \frac{2\alpha\beta}{u^{-1} - (\rho+\tau)} \qquad r_A = \frac{2\alpha^2}{u^{-1} - (\rho+\tau)}$$





only non-resonant quantities

Microscopic interpretation

SPP coupled-mode equations (k_x=0)

$$t_A = t + \frac{2\alpha\beta}{u^{-1} - (\rho + \tau)} \qquad r_A = r + \frac{2\alpha^2}{u^{-1} - (\rho + \tau)}$$

$|u|\approx 1$

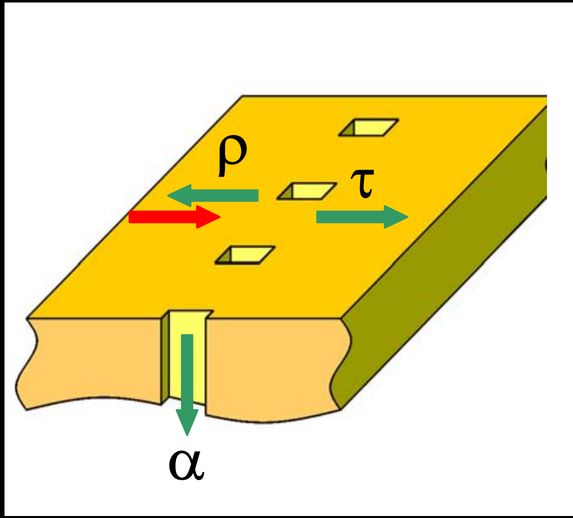
$u=\exp(ik_{SP}a) \rightarrow |u|^{-1}$ slightly larger than 1

$|\tau|\approx 1$

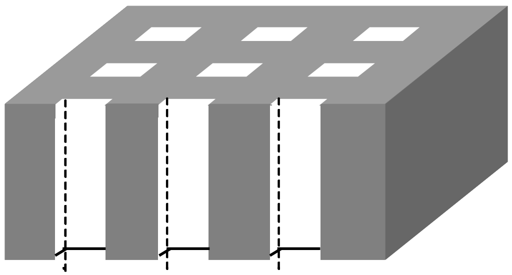
τ slightly smaller than 1

resonance condition

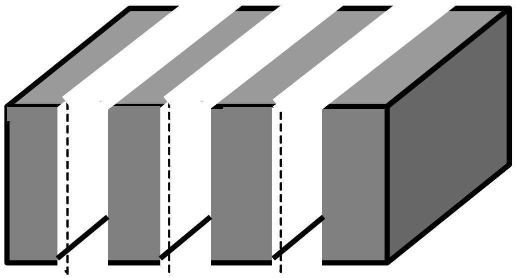
$\mathrm{Re}(k_{SP})a+\arg(\tau) \approx 0$ modulo 2π



holes



slits



$$t_F = \frac{t_A^{\,2}\exp(ik_0 nd)}{1 - r_A^{\,2}\exp(2ik_0 nd)}$$

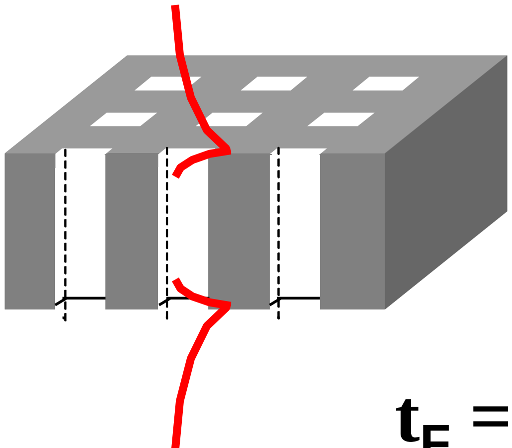
- n complex
- $|\exp(2ik_0nd)| \ll 1$
- pole of t_F = pole of r_A (for $k_0d \gg 1$)

- n real
- $|\exp(2ik_0nd)| = 1$
- no relation between the pole of t_F and that of r_A

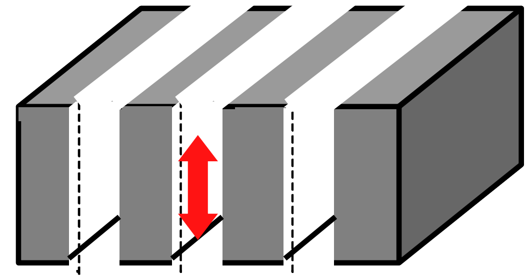
FP condition:

$\arg(r_A) + k_0nd = 2m\pi$

holes



slits



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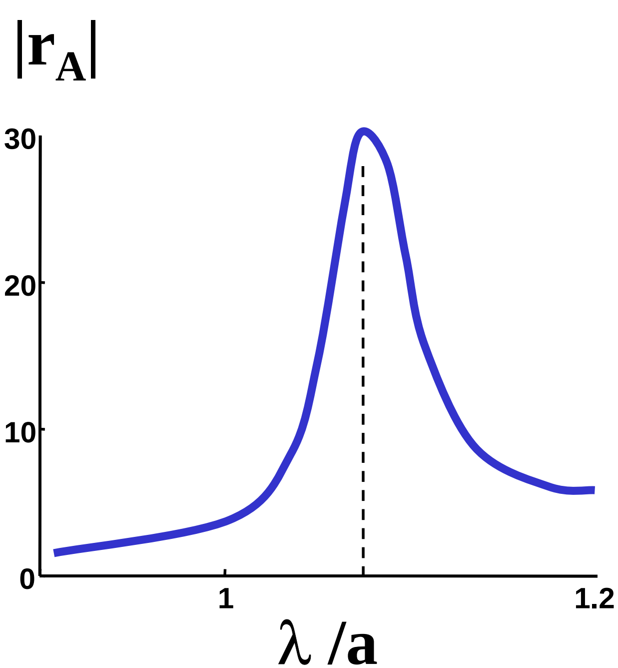

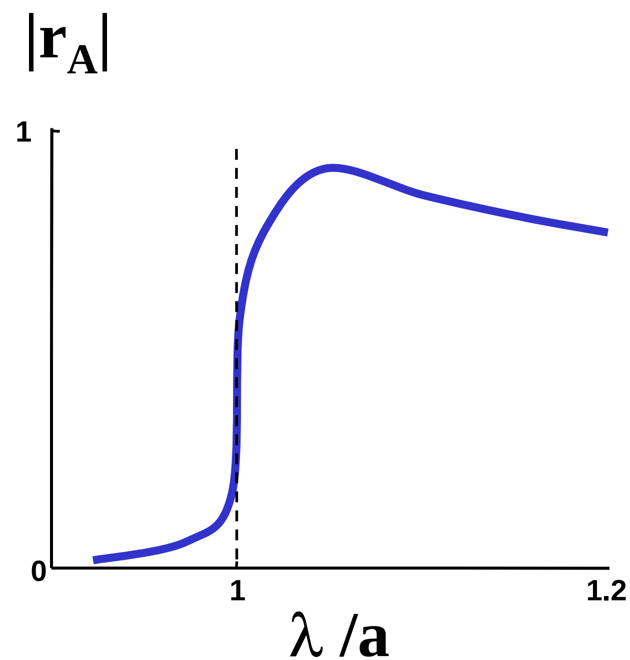

Microscopic interpretation

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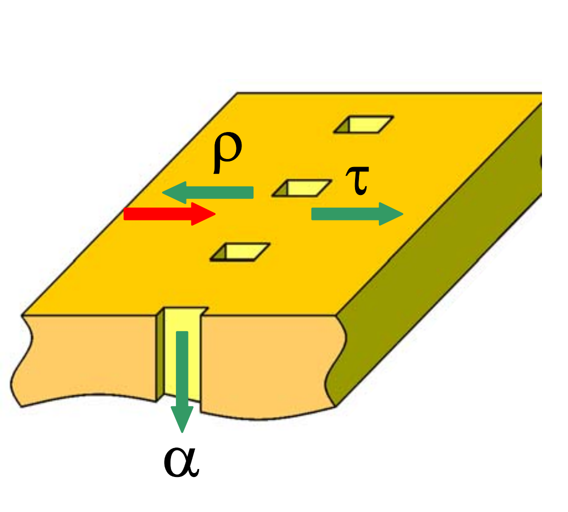
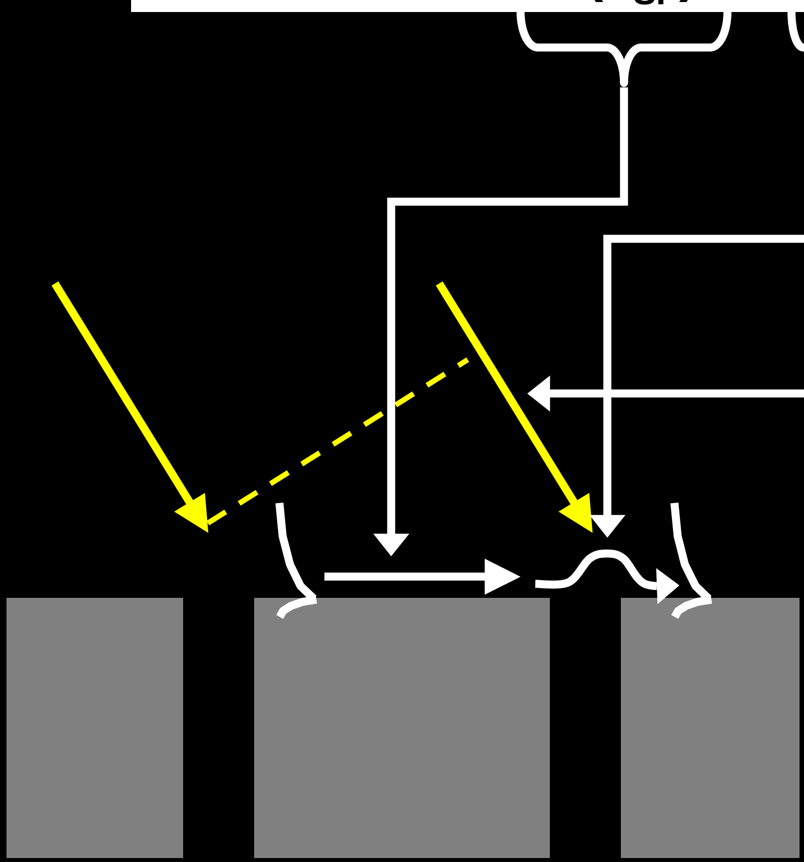


Microscopic interpretation

resonance condition :

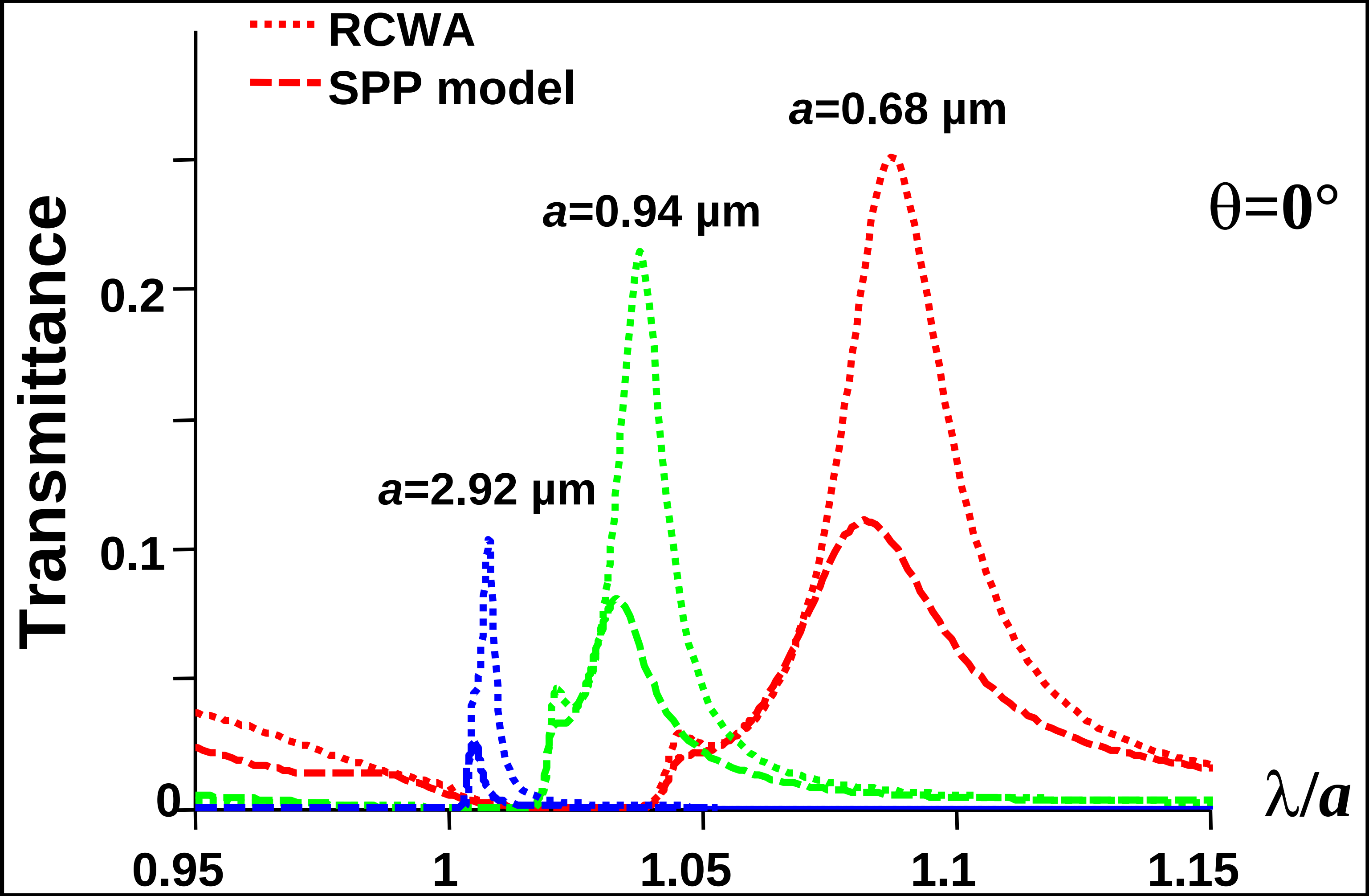
$$\mathrm{Re}(k_{SP})a + \arg(\tau) \approx k_x a \qquad (\text{modulo } 2\pi)$$



the macroscopic surface Bloch mode

superposition of many elementary SPPs scattered by individual hole chains that fly over adjacent chains and sum up constructively

Influence of the metal conductivity



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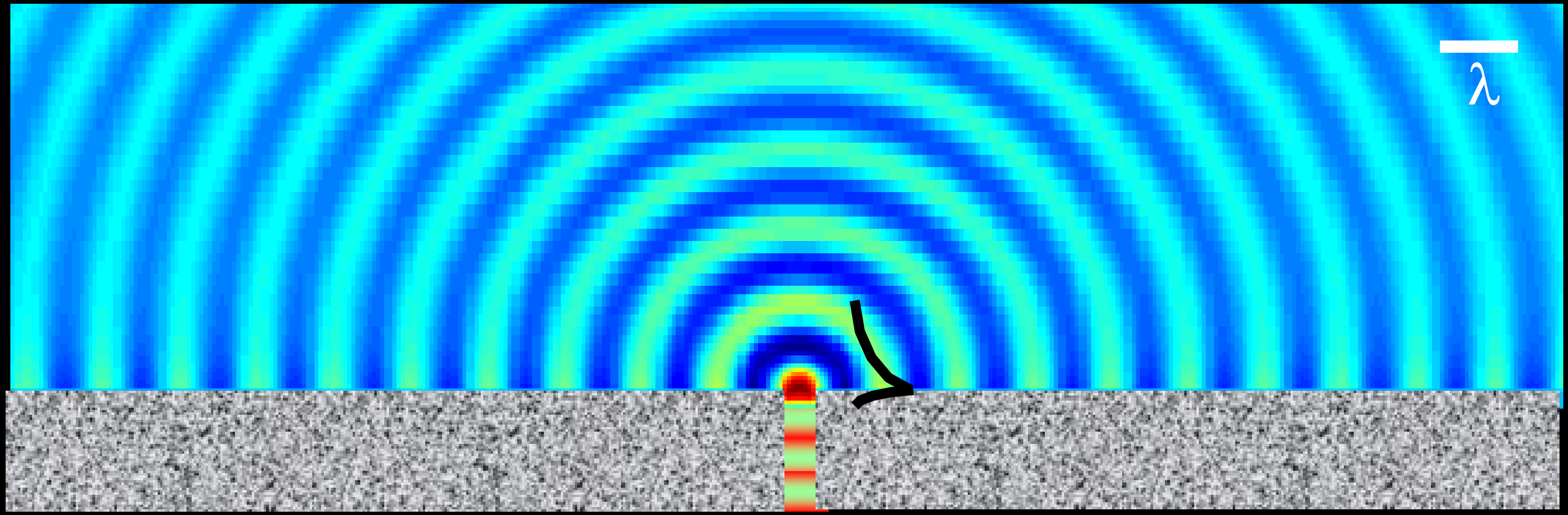
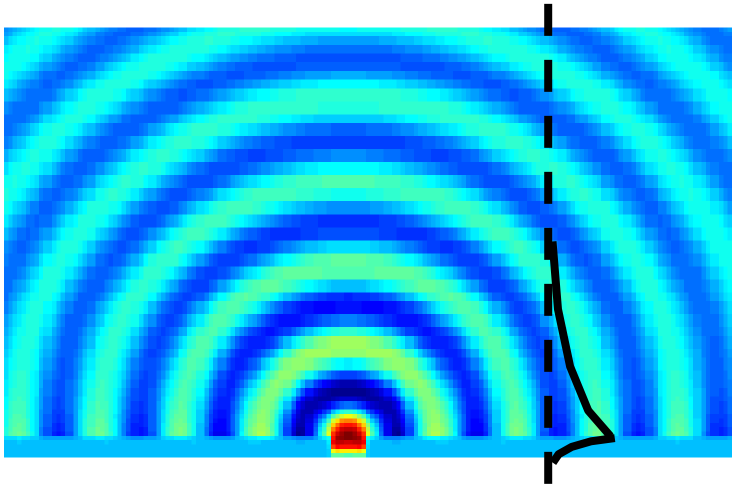


How to know how much SPP is generated?

General theoretical formalism



1) make use of the completeness theorem for the normal modes of waveguides

$$H_y = \left(\alpha^+(x) + \alpha^-(x)\right) H_{SP}(z) + \sum_\sigma a_\sigma(x) H_\sigma^{(rad)}(z)$$

$$E_z = \left(\alpha^+(x) - \alpha^-(x)\right) E_{SP}(z) + \sum_\sigma a_\sigma(x) E_\sigma^{(rad)}(z)$$

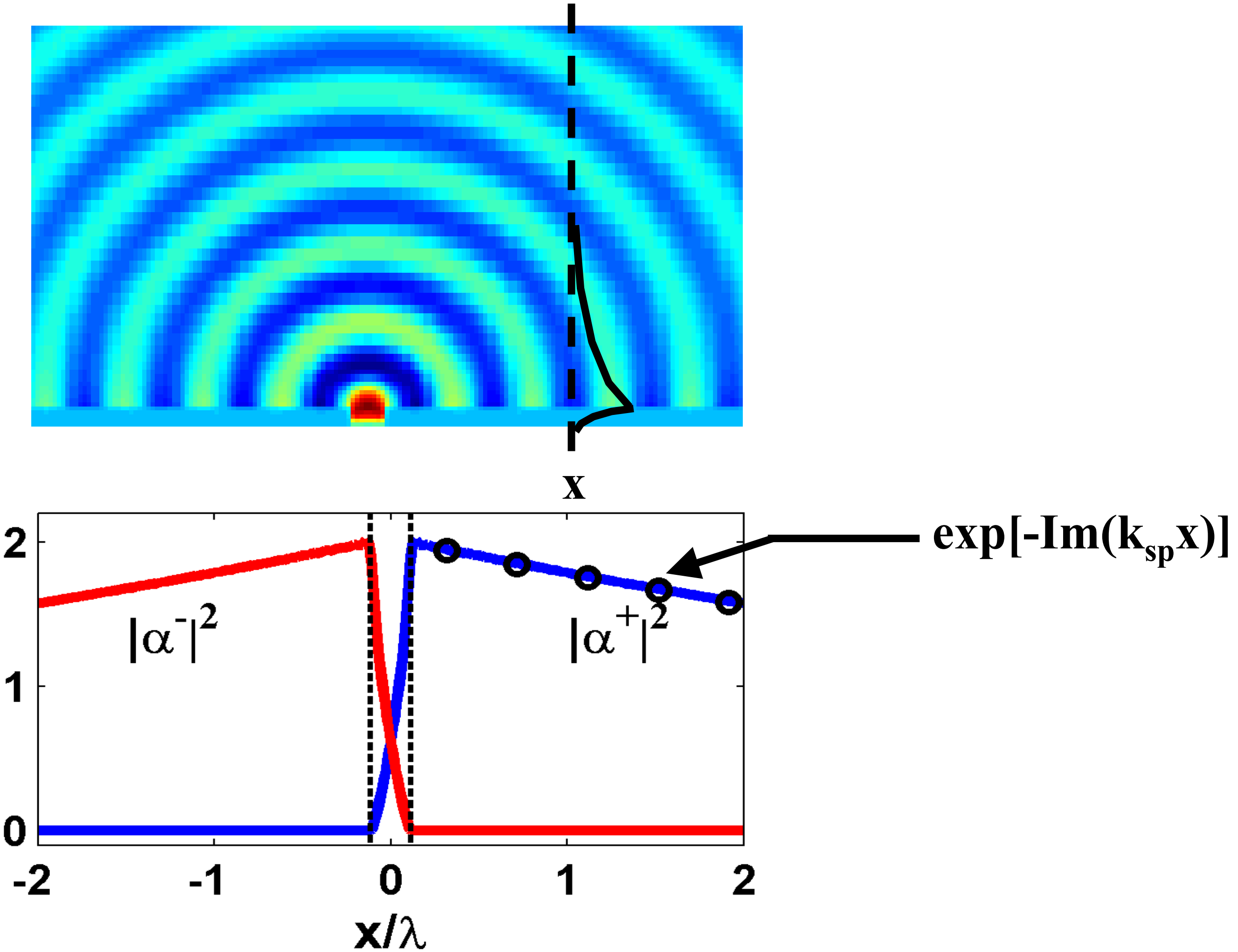
2) Use orthogonality of normal modes

$$\int_{-\infty}^{\infty} dz \;\; H_y(x,z)\, E_{SP}(z) = 2\left(\alpha^+(x) + \alpha^-(x)\right)$$

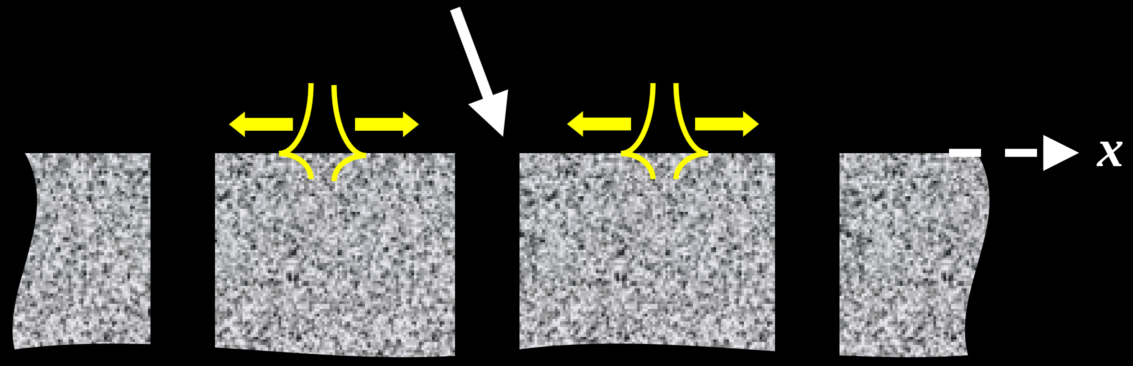
$$\int_{-\infty}^{\infty} dz \;\; E_z(x,z)\, H_{SP}(z) = 2\left(\alpha^+(x) - \alpha^-(x)\right)$$

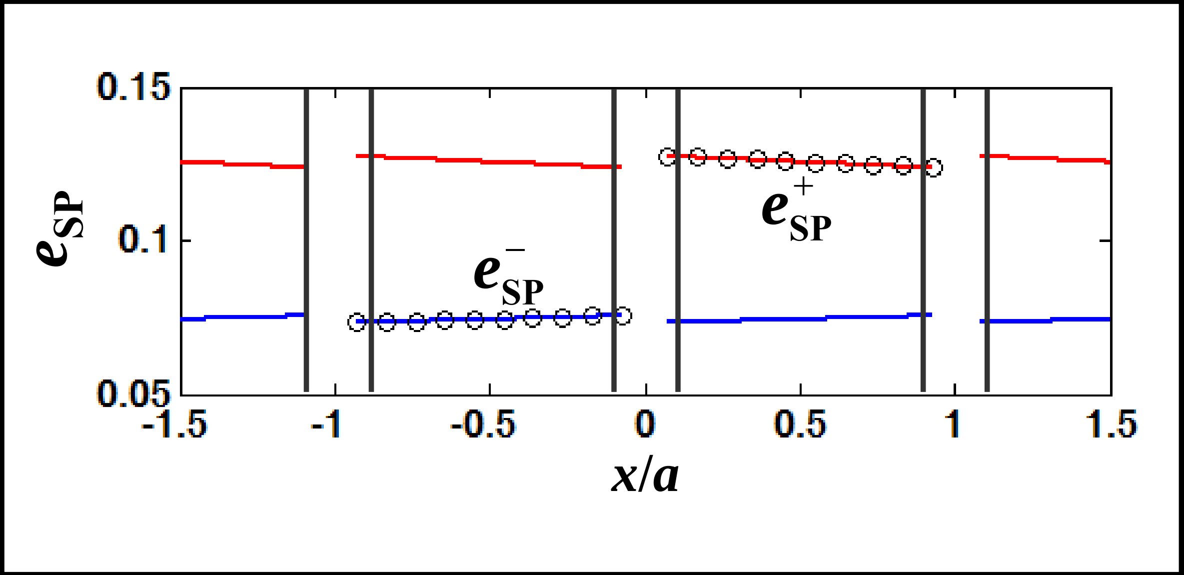
PL, J.P. Hugonin and J.C. Rodier, PRL 95, 263902 (2005)

General theoretical formalism

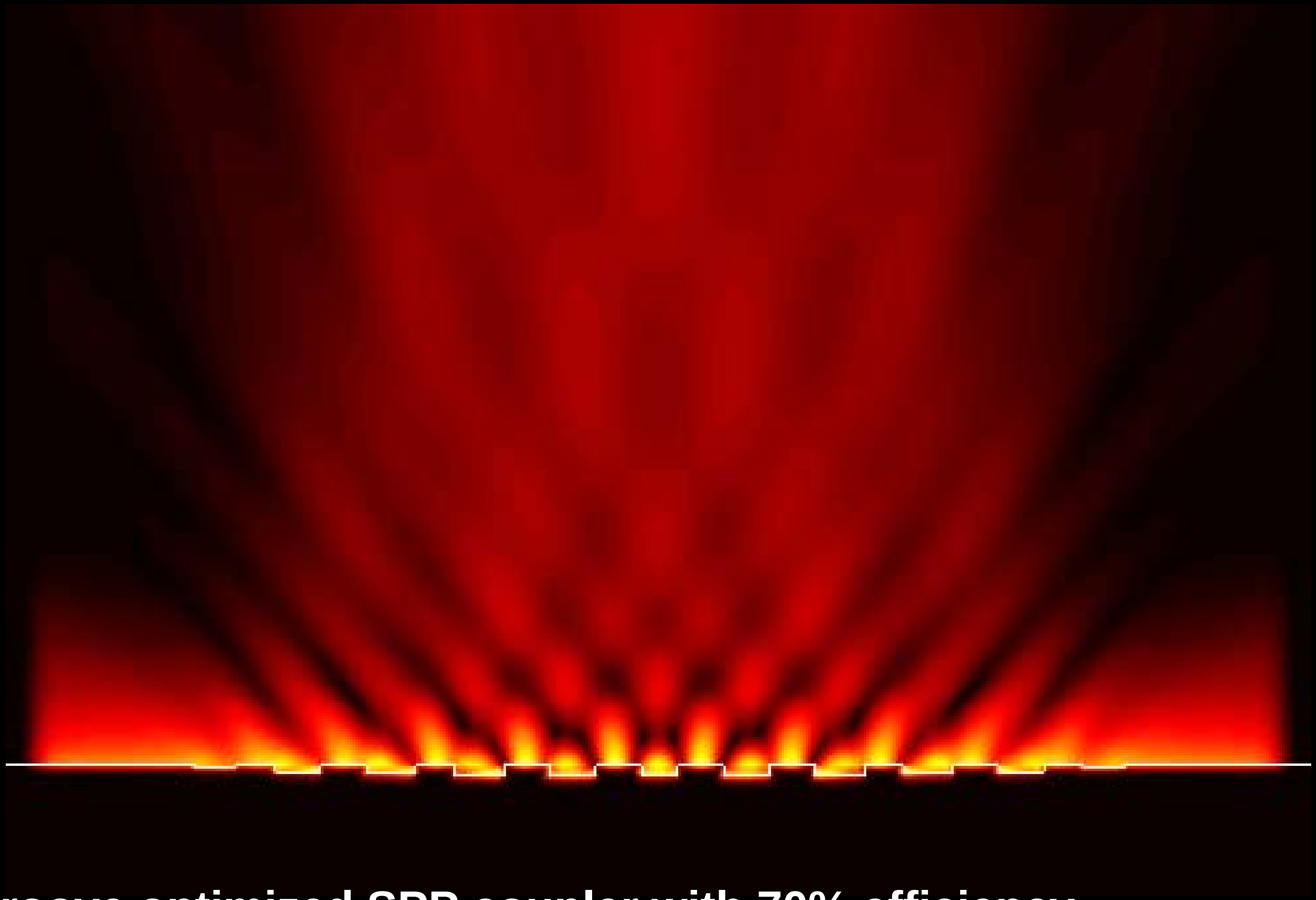


General theoretical formalism applied to gratings



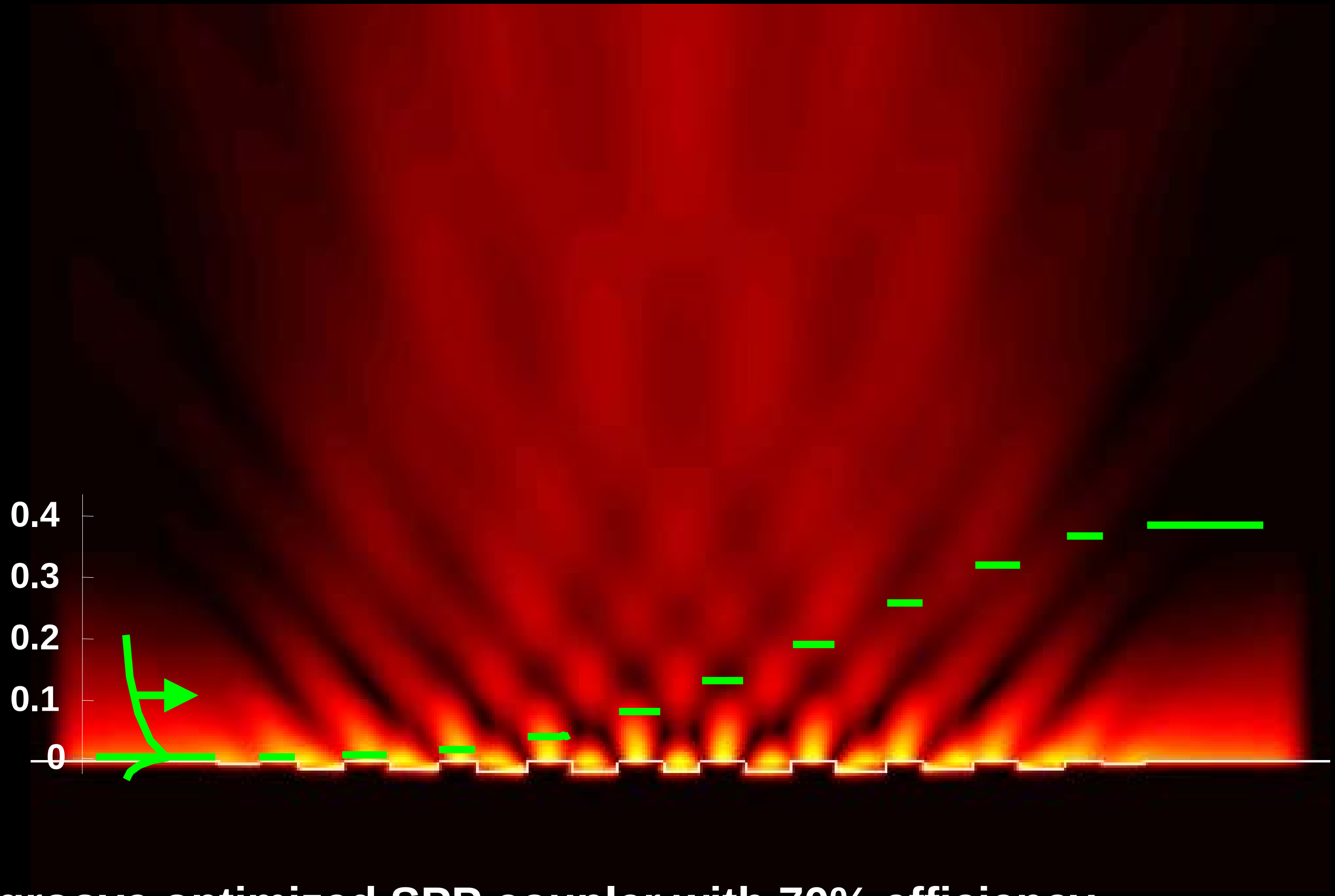


General theoretical formalism applied to grooves ensembles



- 11-groove optimized SPP coupler with 70% efficiency.
- The incident gaussian beam has been removed.

General theoretical formalism



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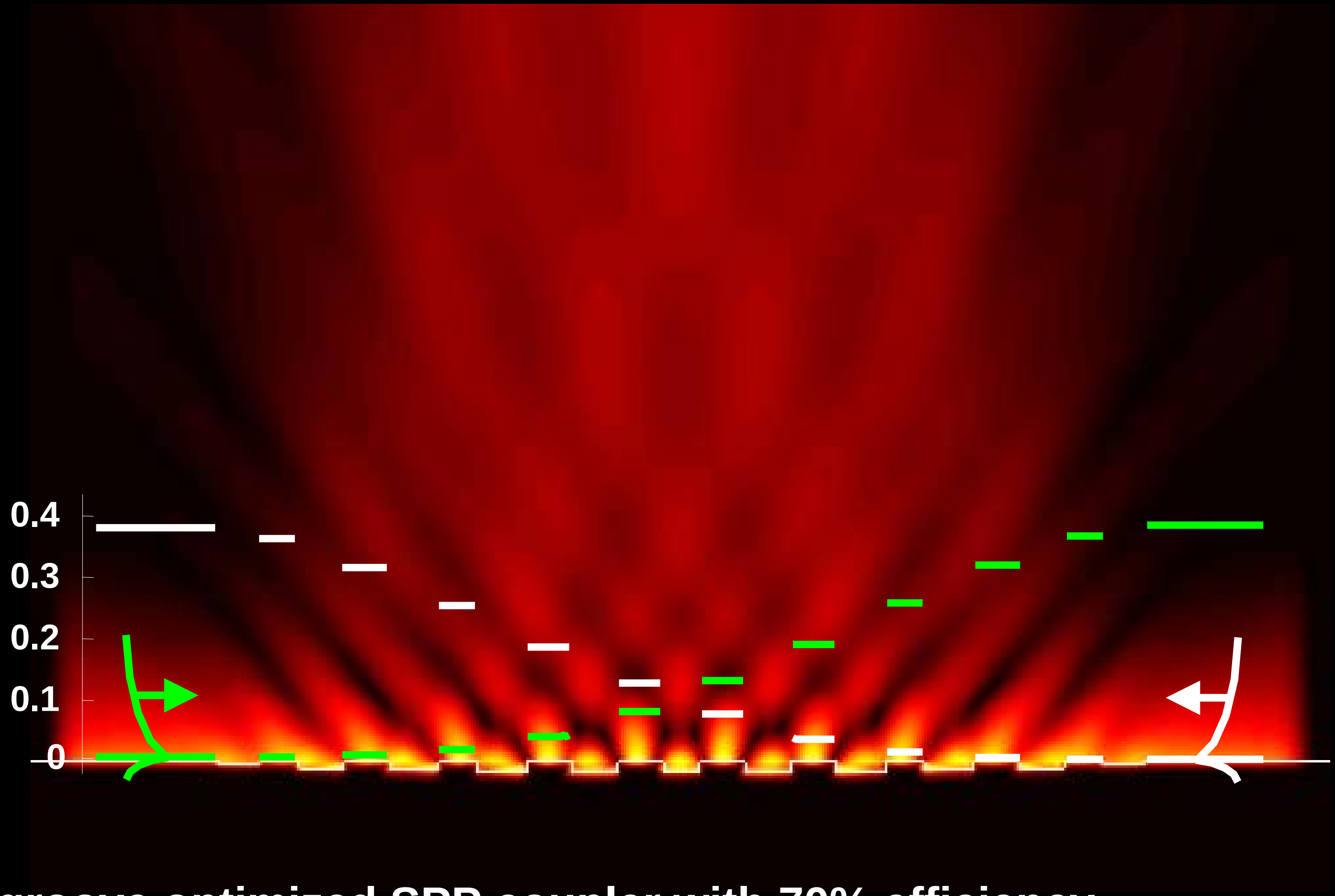

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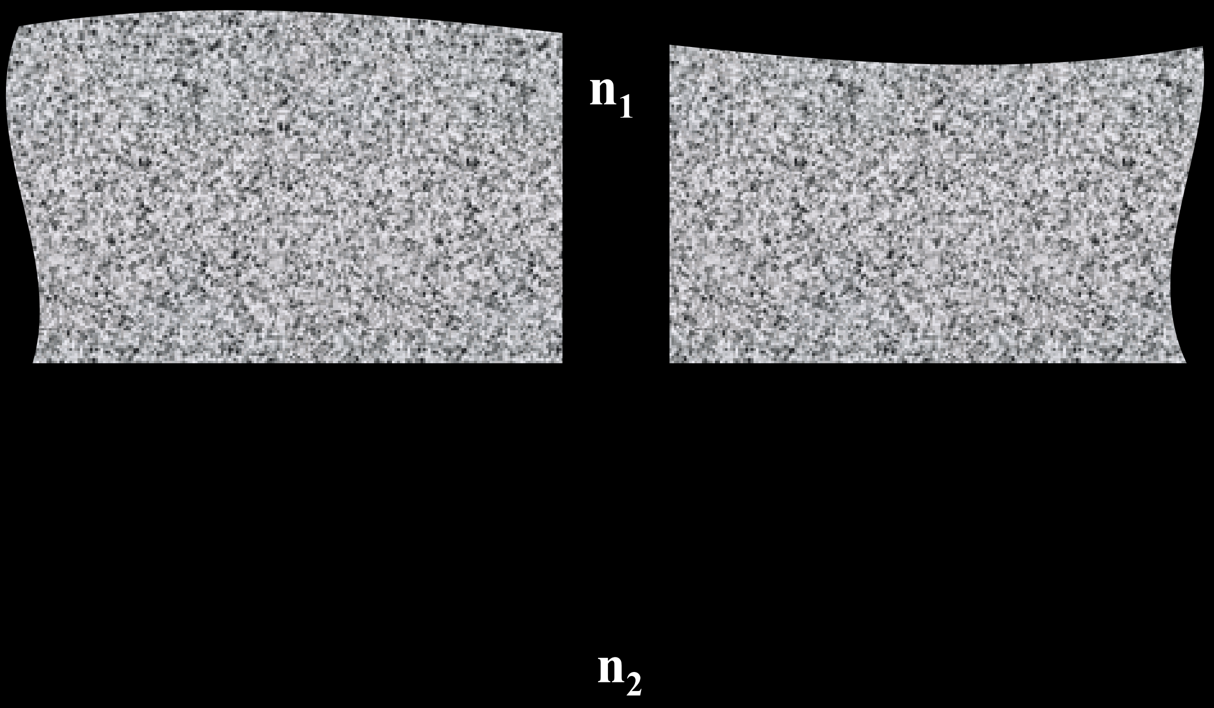
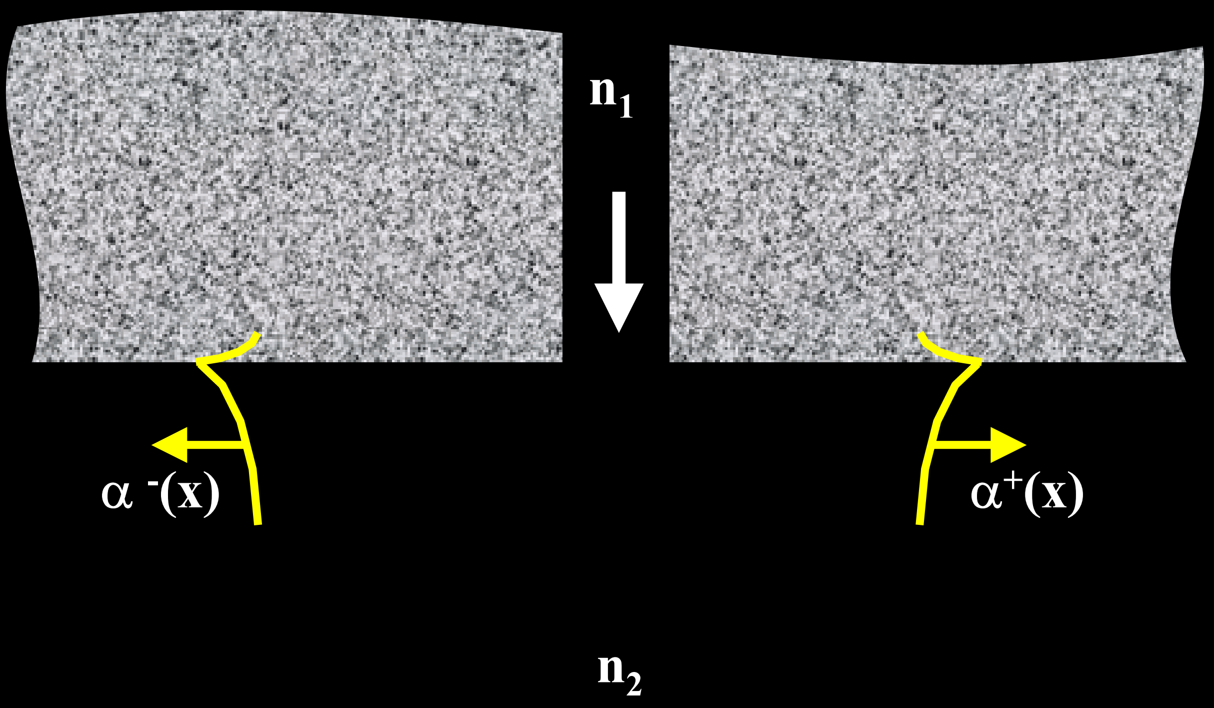
SPP generation



SPP generation



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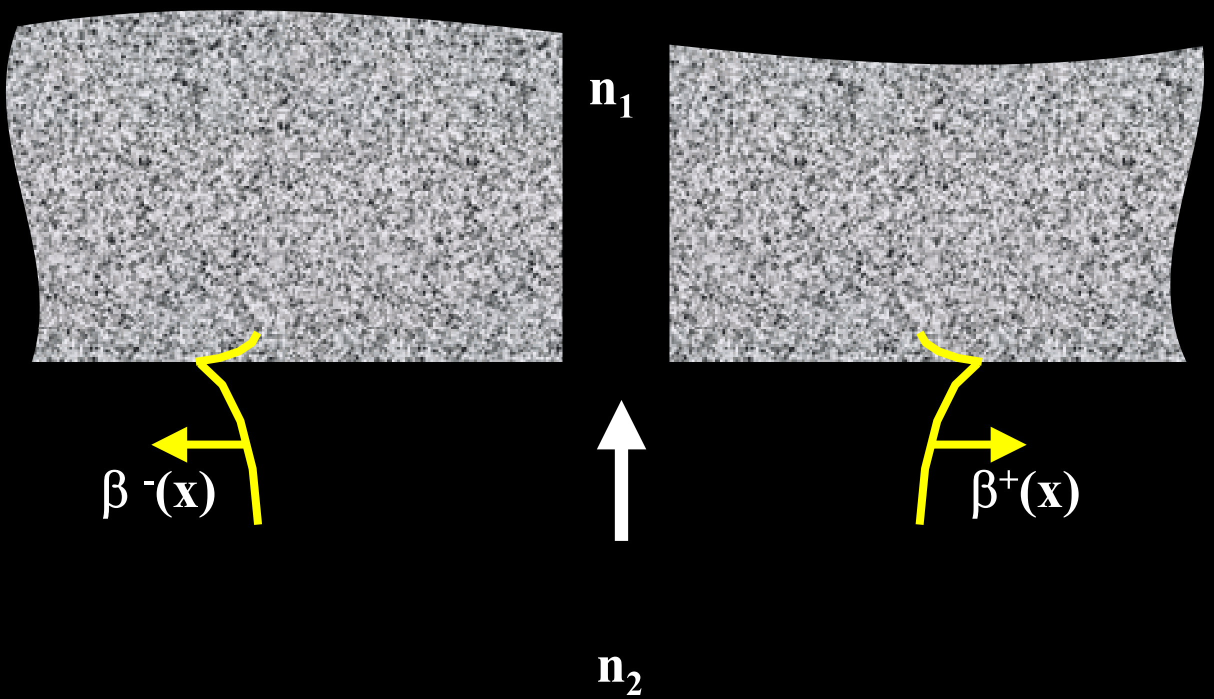
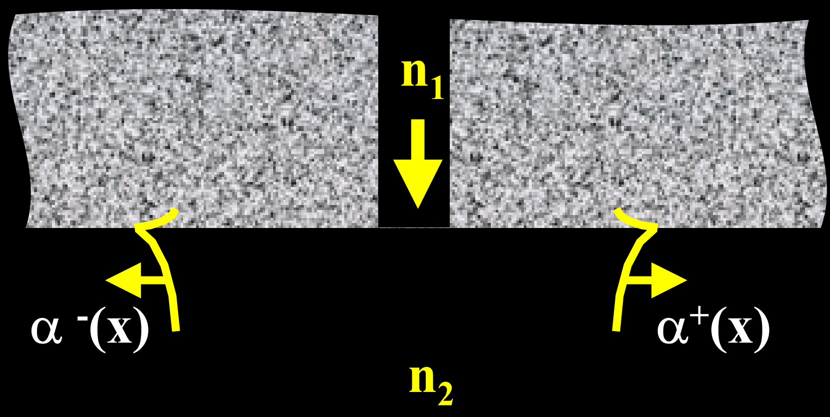


Analytical model



1) assumption : the near-field distribution in the immediate vicinity of the slit is weakly dependent on the dielectric properties

2) Calculate this field for the PC case

3) Use orthogonality of normal modes

$$\int_{-\infty}^{\infty} dz\ H_y(x,z)\, E_{SP}(z) = 2\left(\alpha^+(x) + \alpha^-(x)\right)$$

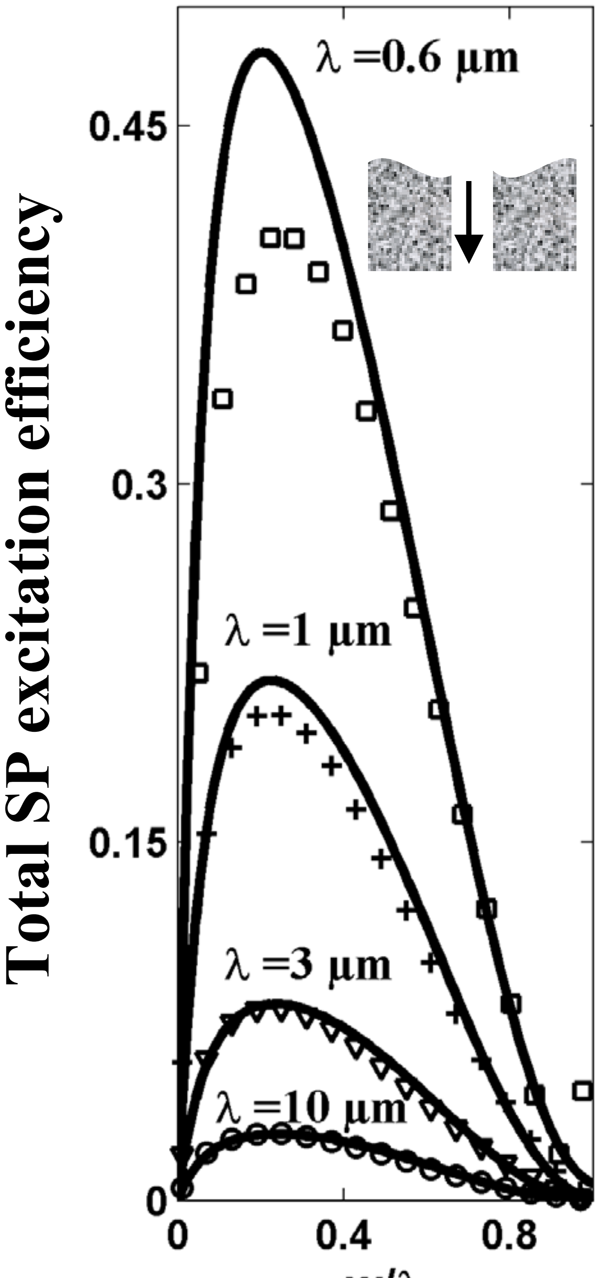
$$\int_{-\infty}^{\infty} dz\ E_z(x,z)\, H_{SP}(z) = 2\left(\alpha^+(x) - \alpha^-(x)\right)$$

Valid only at x = ±*w*/2 only!

$$\alpha^+ = \alpha^- = \left(\frac{4}{\pi}\frac{n_2}{n_1}\frac{\sqrt{|\varepsilon|}}{\varepsilon + n_2^2}\right)^{1/2} \frac{\sqrt{\frac{w}{\lambda}}\, I_1}{1 + \left(n_2/n_1\right) w'\, I_0}$$

PL, J.P. Hugonin and J.C. Rodier, JOSAA 23, 1608 (2006).

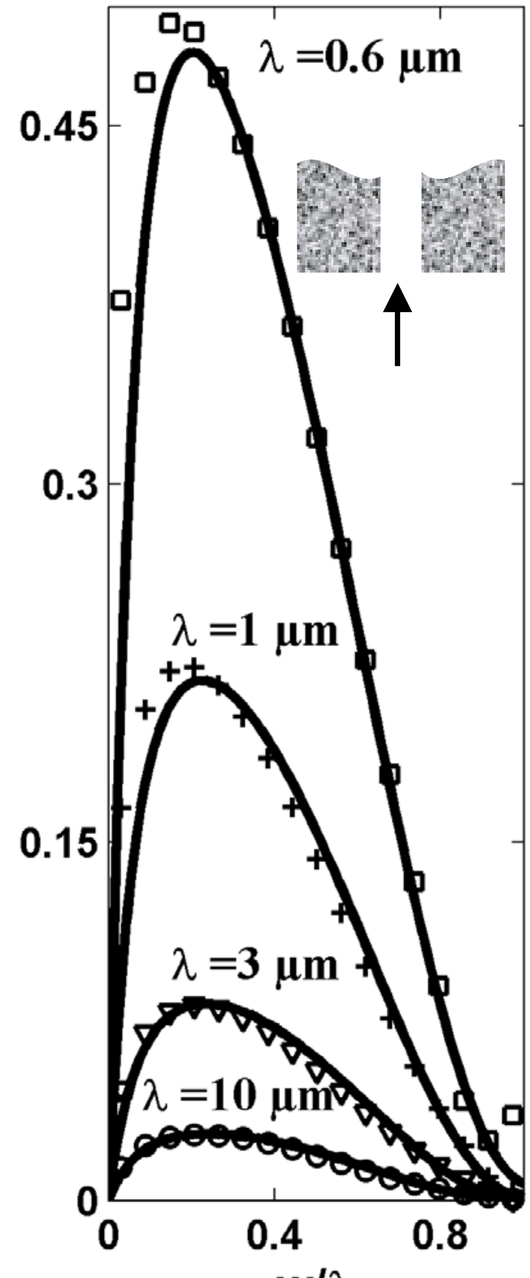
$|\alpha|^2$ results obtained for gold $|\beta|^2$



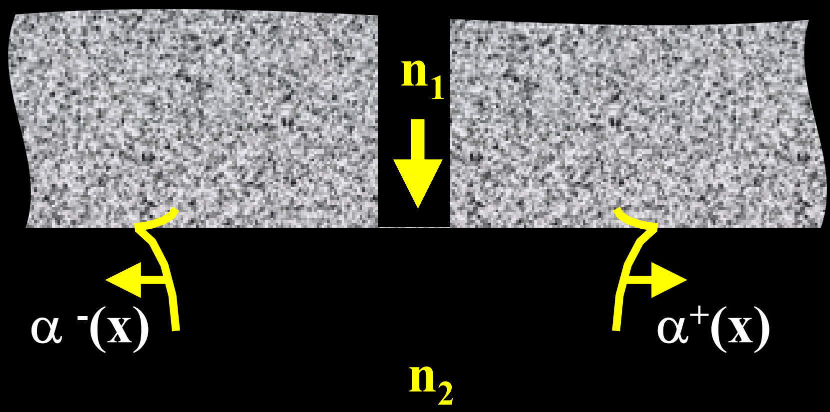
solid curves (model)

marks (calculation)

Question: how is it possible that with a perfect metallic case, ones gets accurate results?



Analytical model



1) assumption : the near-field distribution in the immediate vicinity of the slit is weakly dependent on the dielectric properties

2) Calculate this field for the PC case

3) Use orthogonality of normal modes

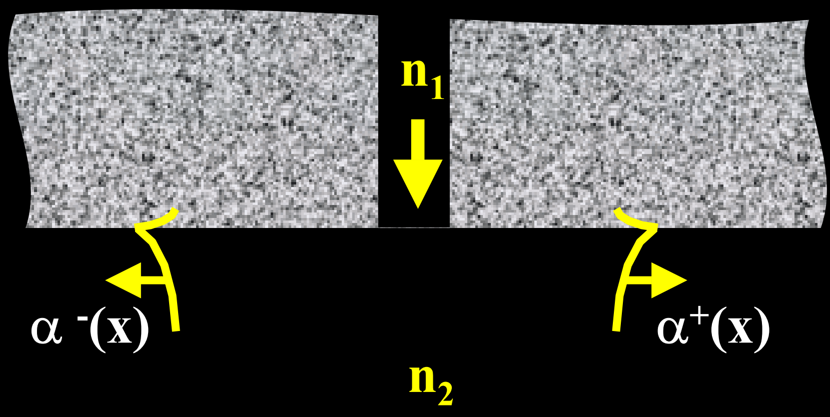
$$\alpha^+ = \alpha^- = \left(\frac{4}{\pi} \frac{n_2}{n_1} \frac{\sqrt{|\varepsilon|}}{\varepsilon + n_2^2} \right)^{1/2} \frac{\sqrt{\frac{w}{\lambda}} I_1}{1 + (n_2/n_1) w' I_0}$$

describe geometrical properties

-the SPP excitation peaks at a value *w*=0.23λ and for visible frequency, $|\alpha|^2$ can reach 0.5, which means that of the power coupled out of the slit half goes into heat

EXP. VERIFICATION : H.W. Kihm et al., "Control of surface plasmon efficiency by slit-width tuning", APL 92, 051115 (2008) & S. Ravets et al., "Surface plasmons in the Young slit doublet experiment", JOSA B 26, B28 (special issue plasmonics 2009).

Analytical model



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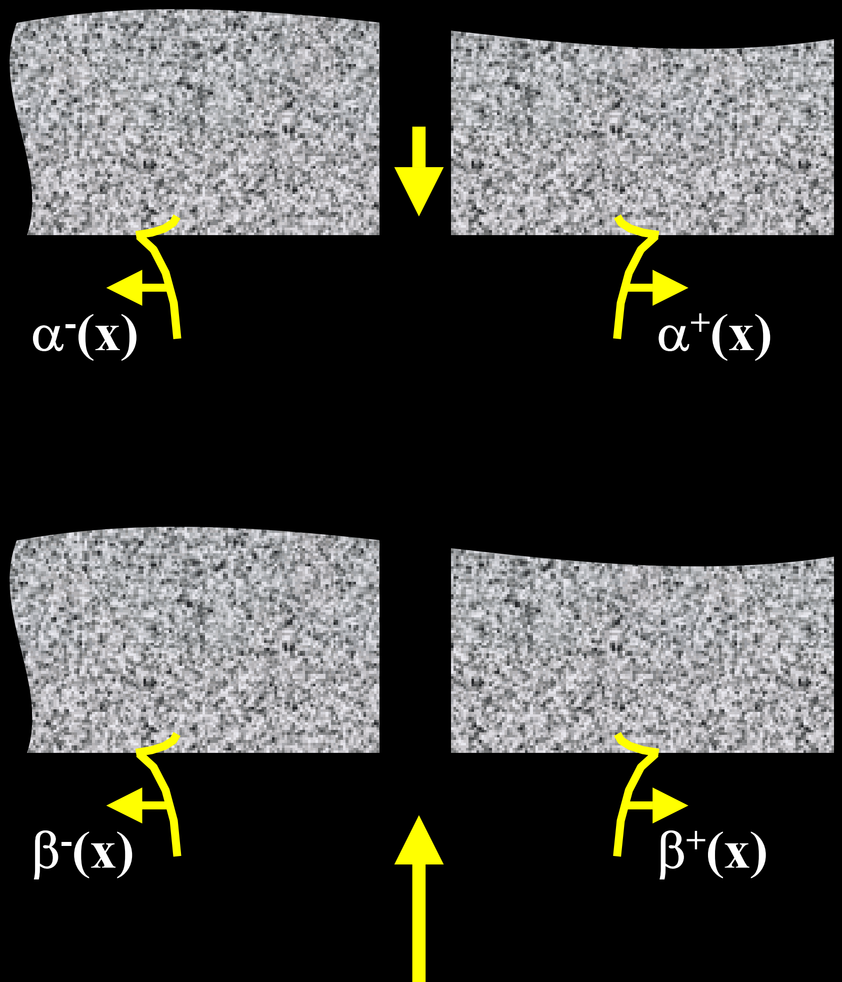
describe material properties

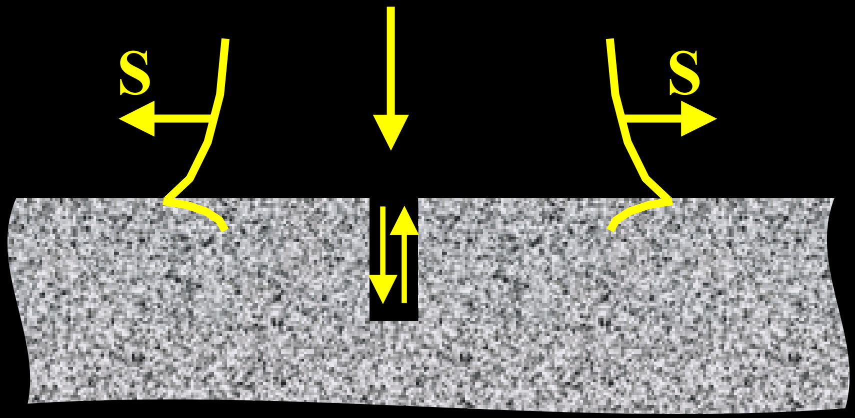
-Immersing the sample in a dielectric enhances the SP excitation ($\propto n_2/n_1$)

-The SP excitation probability $|\alpha^+|^2$ scales as $|\varepsilon(\lambda)|^{-1/2}$

PL, J.P. Hugonin and J.C. Rodier, JOSAA 23, 1608 (2006).


$\alpha^-(\mathbf{x})$
$\alpha^+(\mathbf{x})$
$\beta^-(\mathbf{x})$
$\beta^+(\mathbf{x})$

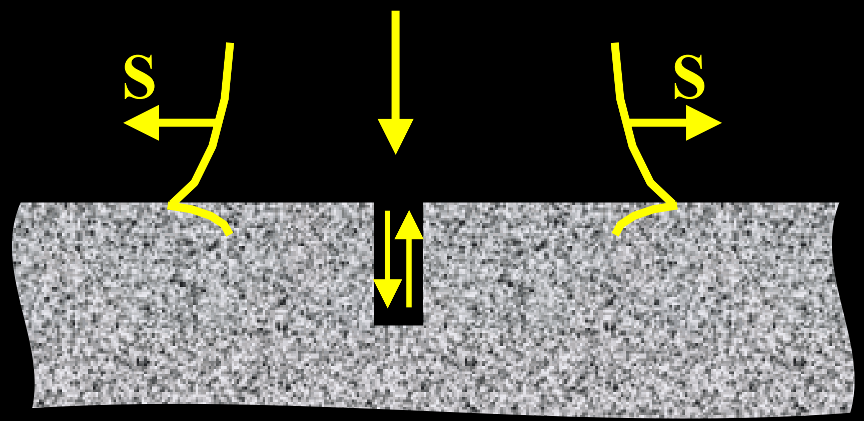
Grooves



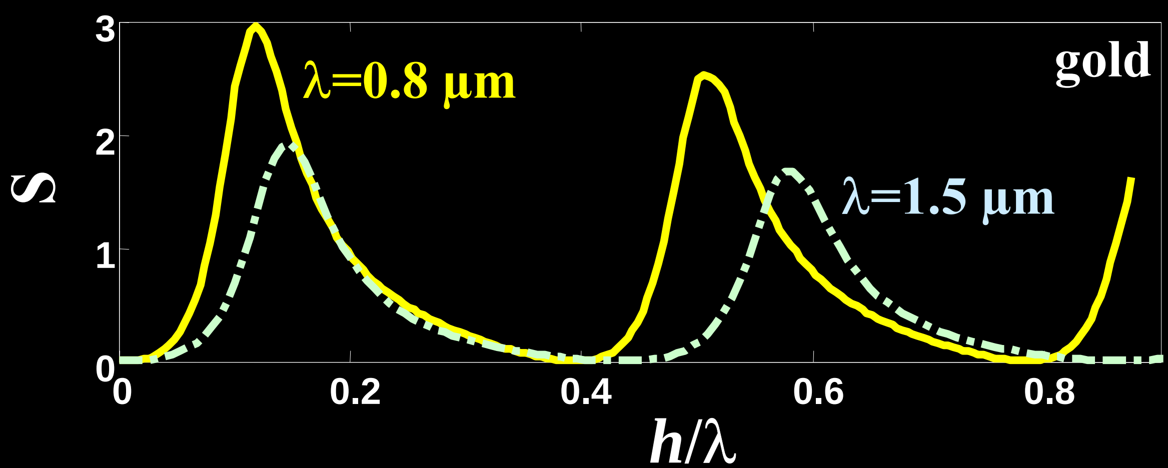
$$S = \beta + [t_0\ \alpha\ \exp(2ik_0 n_{eff} h)]\ /\ [1 - r_0\ \exp(2ik_0 n_{eff} h)]$$

PL, J.P. Hugonin and J.C. Rodier, JOSAA 23, 1608 (2006).

groove

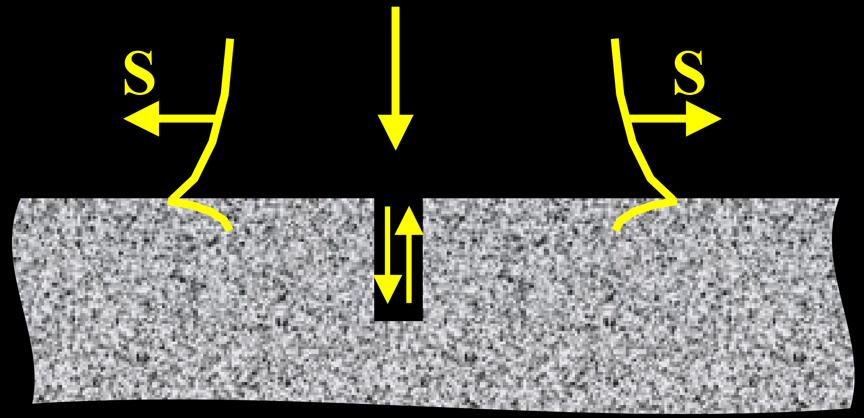


$$S = \beta + [t_0 \, \alpha \, \exp(2ik_0n_{eff}h)] \, / \, [1 - r_0 \exp(2ik_0n_{eff}h)]$$

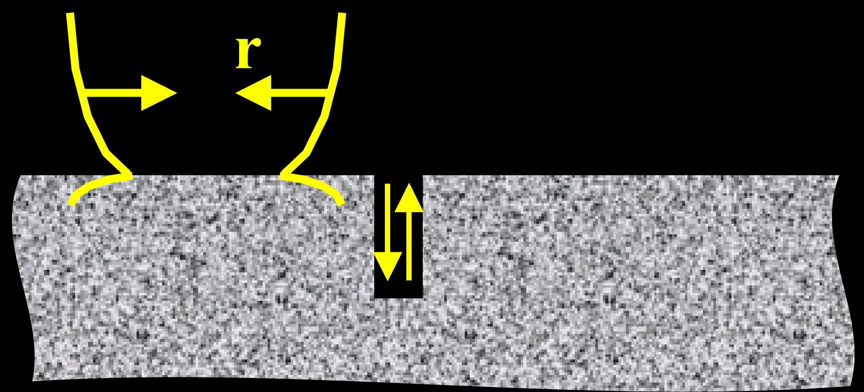


Can be much larger than the geometric aperture!

Grooves



$$S = \beta + [t_0\, \alpha\, \exp(2ik_0 n_{eff} h)] / [1 - r_0 \exp(2ik_0 n_{eff} h)]$$



$$r = r_\infty + \alpha^2 \exp(2ik_0 n_{eff} h)] / [1 - r_0 \exp(2ik_0 n_{eff} h)]$$

PL, J.P. Hugonin and J.C. Rodier, JOSAA 23, 1608 (2006).

1.Why a microscopic analysis?

-extraodinary optical transmission (EOT)
-limitation of classical "macroscopic" grating theories
-a microscopic pure-SPP model of the EOT

2.SPP generation by sub-λ indentation

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-the important slit example
-scaling law with the wavelength

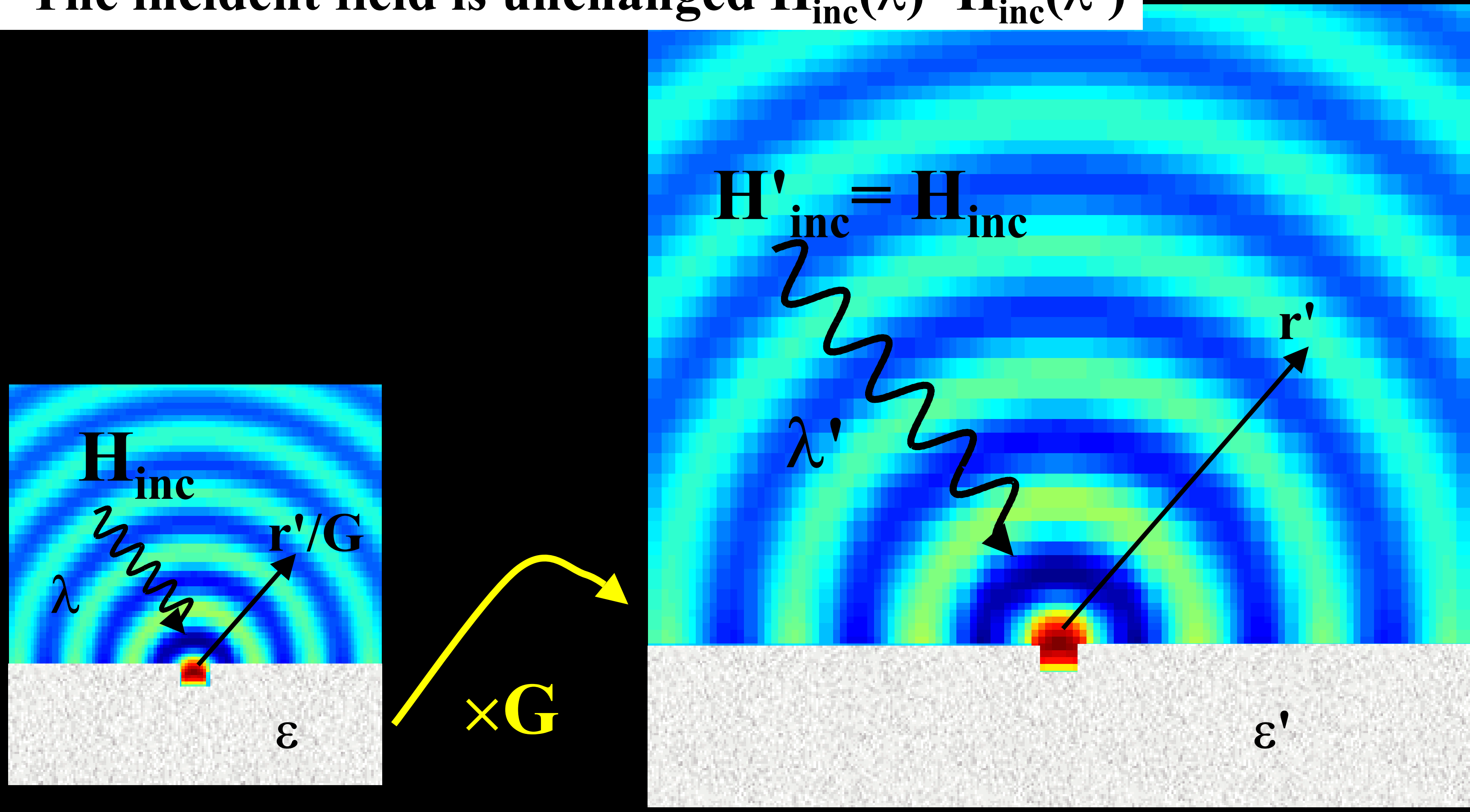
3.The quasi-cylindrical wave

-definition & properties
-scaling law with the wavelength

4.Multiple Scattering of SPPs & quasi-CWs

-definition of scattering coefficients for the quasi-CW

- All dimensions are scaled by a factor G
- The incident field is unchanged $H_{inc}(\lambda)=H_{inc}(\lambda')$



If ε=ε', the scattered field is unchanged : $E'(r')=E(r'/G)$

The difficulty comes from the dispersion : $\varepsilon' \neq \varepsilon$

$\varepsilon'/\varepsilon = G^2$ (for Drude metals)

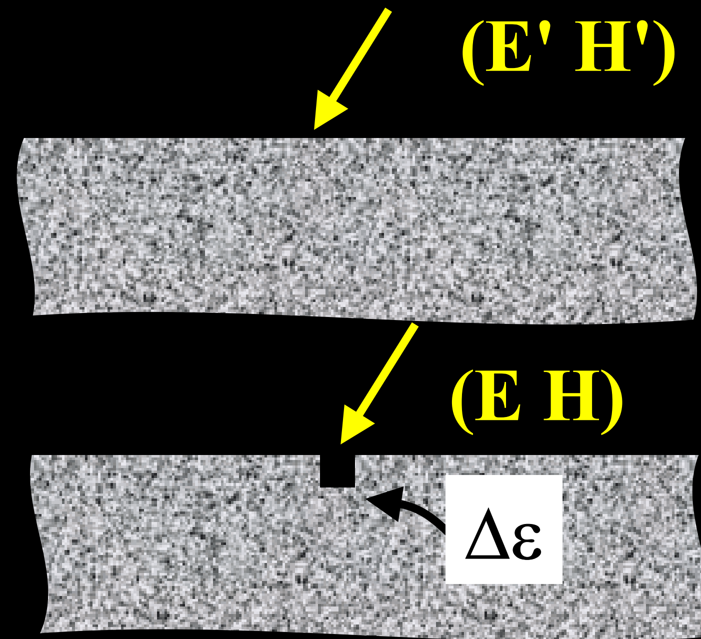
A perturbative analysis fails

Unperturbed system

$$\begin{cases} \nabla\times \mathbf{E}' = j\omega\mu_0\mathbf{H}' \\ \nabla\times\mathbf{H}' = -j\omega\varepsilon(\mathbf{r})\mathbf{E}' \end{cases}$$

Perturbed system

$$\begin{cases} \nabla\times\mathbf{E} = j\omega\mu_0\mathbf{H} \\ \nabla\times\mathbf{H} = -j\omega\varepsilon(\mathbf{r})\mathbf{E} - j\omega\Delta\varepsilon\mathbf{E} \end{cases}$$

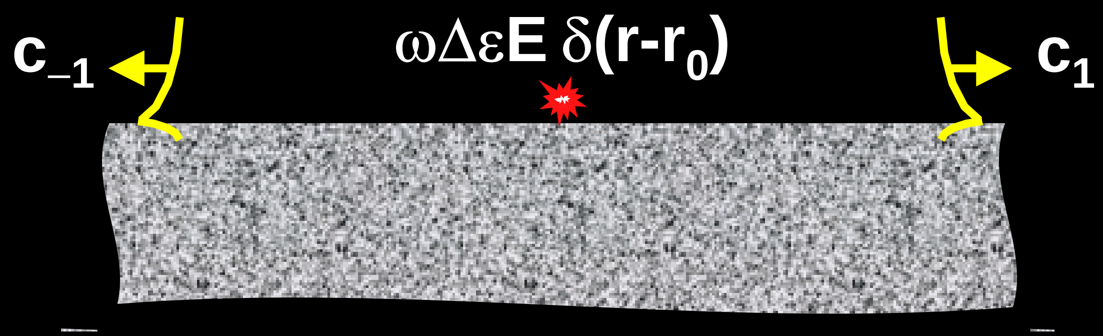


$$\mathbf{E}_s=\mathbf{E}-\mathbf{E}' \;\&\; \mathbf{H}_s=\mathbf{H}-\mathbf{H}'$$

$$\begin{cases} \nabla\times\mathbf{E}_s = j\omega\mu_0\mathbf{H}_s \\ \nabla\times\mathbf{H}_s = -j\omega\varepsilon(\mathbf{r})\mathbf{E}_s - j\omega\Delta\varepsilon\mathbf{E} \end{cases}$$

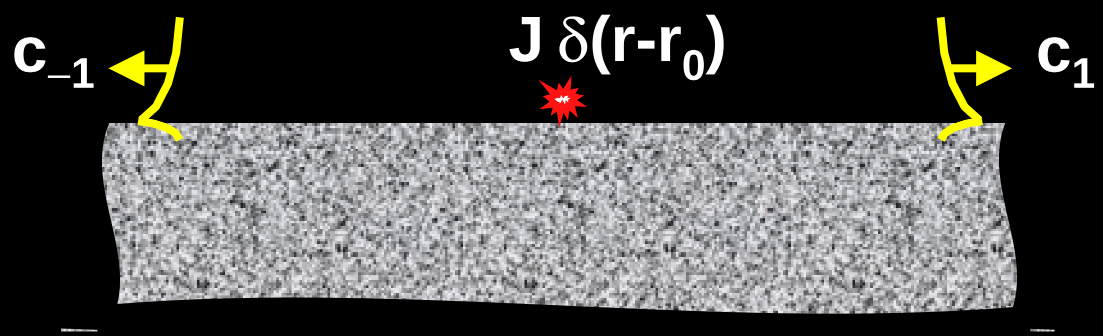
Indentation acts like a volume current source : $\mathbf{J} = -j\omega\Delta\varepsilon\mathbf{E}$

A perturbative analysis fails



One may find how c_1 or c_{-1} scale.

"Easy" task with the reciprocity



Source-mode reciprocity

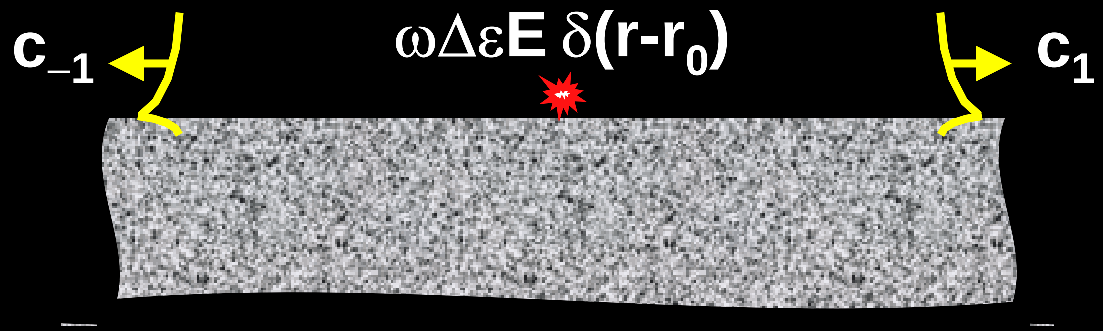
$$c_1 = -E_{SP}^{(-1)}(r_0) \cdot J$$
$$c_{-1} = -E_{SP}^{(1)}(r_0) \cdot J$$

provided that the SPP pseudo-Poynting flux is normalized to 1

$$\tfrac{1}{2} \int dz\, [E_{SP}^{(1)} \times H_{SP}^{(1)})] \cdot x = 1$$

$E_{SP} = N^{1/2} \exp(ik_{SP}x)\exp(i\gamma_{SP}z)$, **with** $N \approx |\varepsilon|^{1/2}/(4\omega\varepsilon_0)$

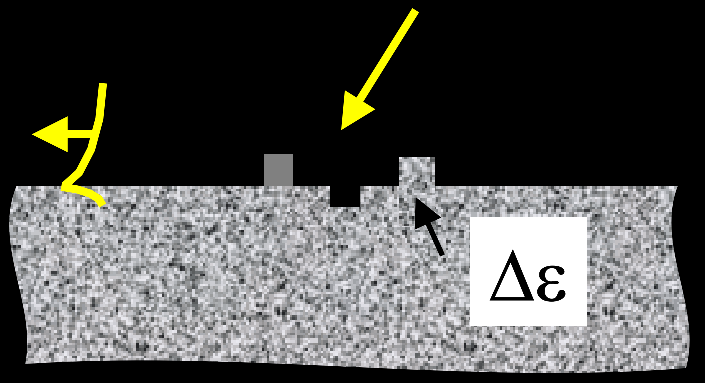
A perturbative analysis fails



One may find how c_1 or c_{-1} scale.

Then, one may apply the first-order Born approximation ($E = E' \approx E_{inc}/2$).

Since $E_{inc} \sim 1$, one obtains that dielectric and metallic indentations scale differently.

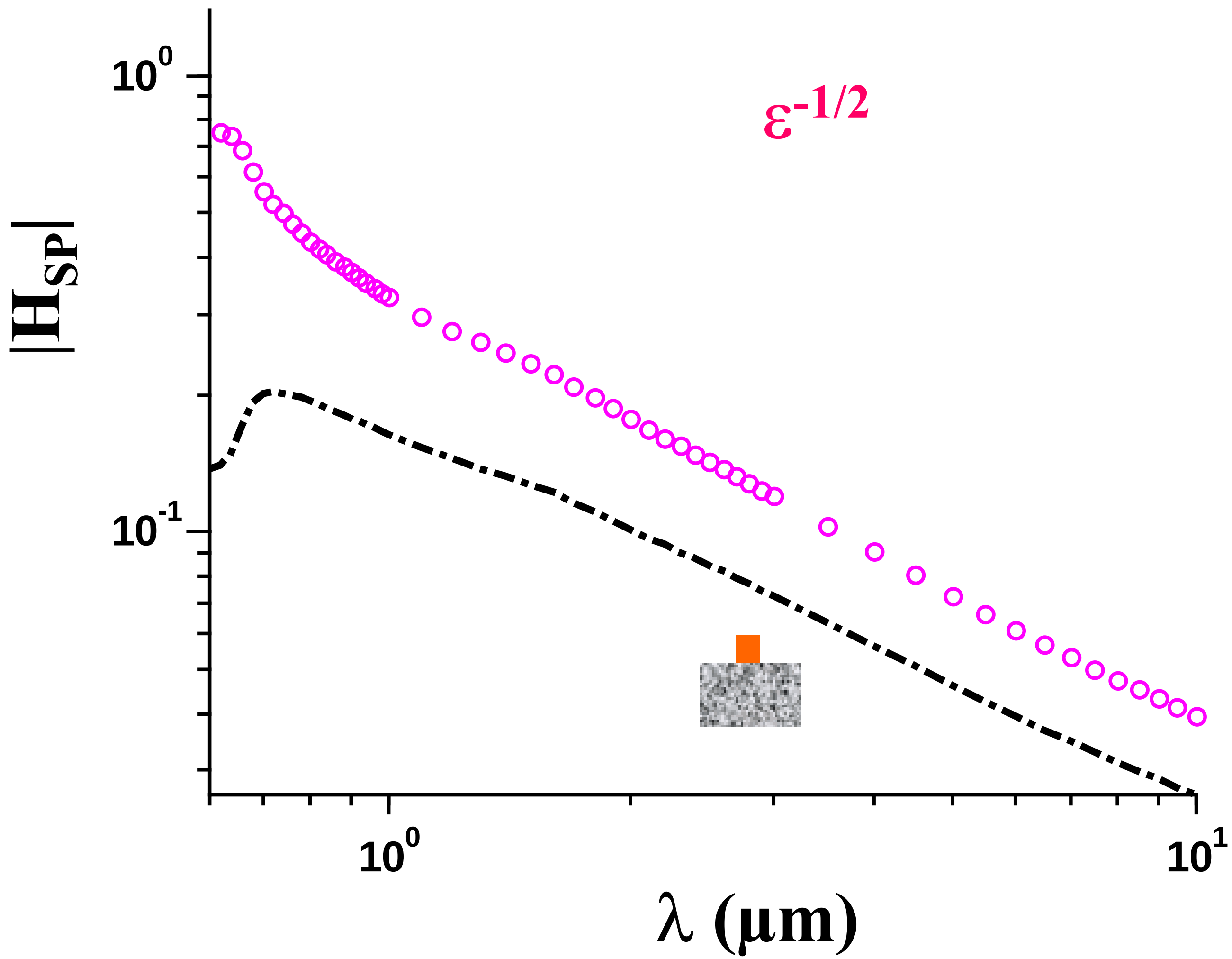


$\Delta\varepsilon \sim 1$ for dielectric ridges

$\Delta\varepsilon \sim \varepsilon$ for dielectric grooves

$\Delta\varepsilon \sim \varepsilon$ for metallic ridges

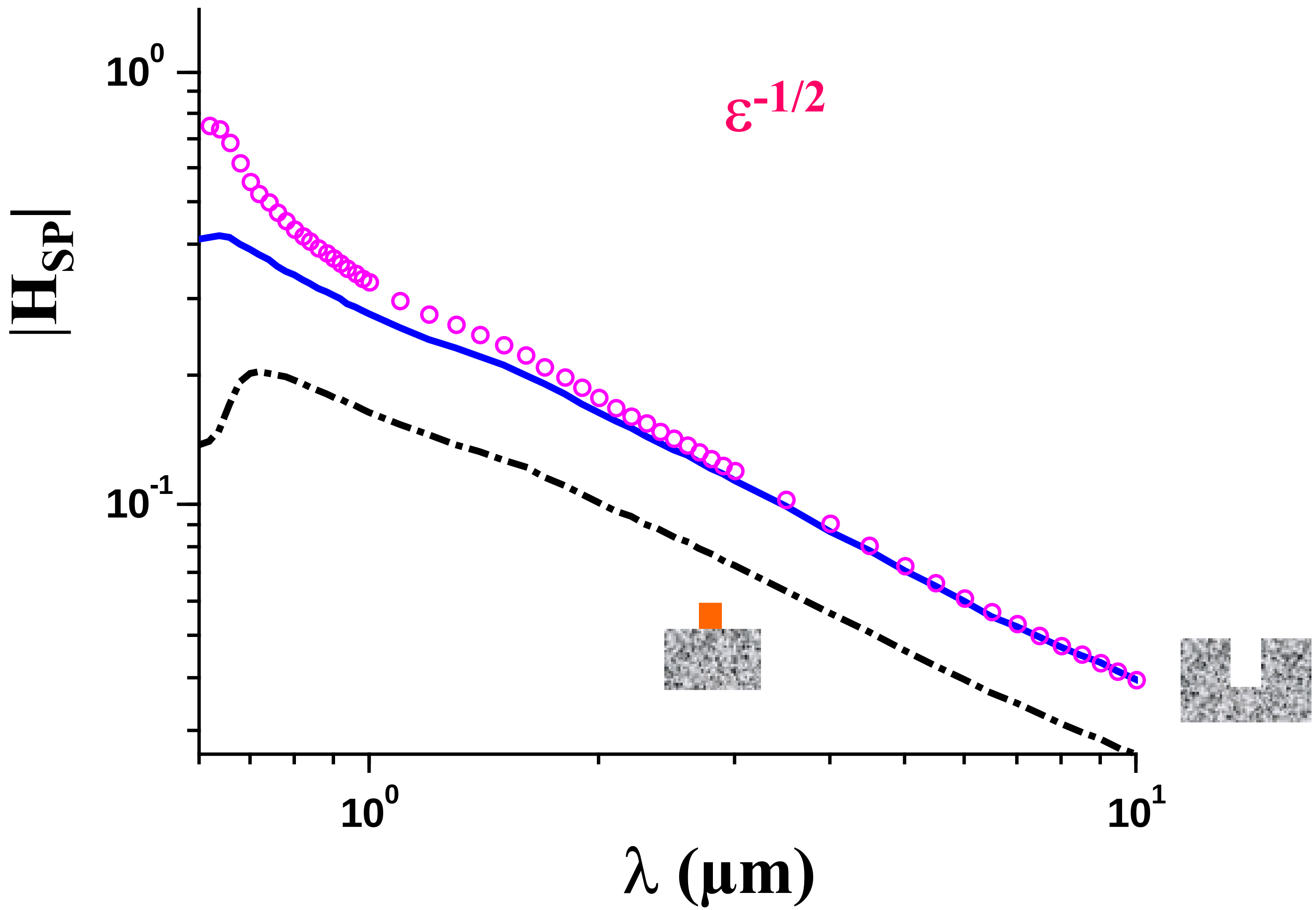
Numerical verification



H. Liu et al., IEEE JSTQE 14, 1522 (2009 special issue)

(results obtained for gold)

Numerical verification



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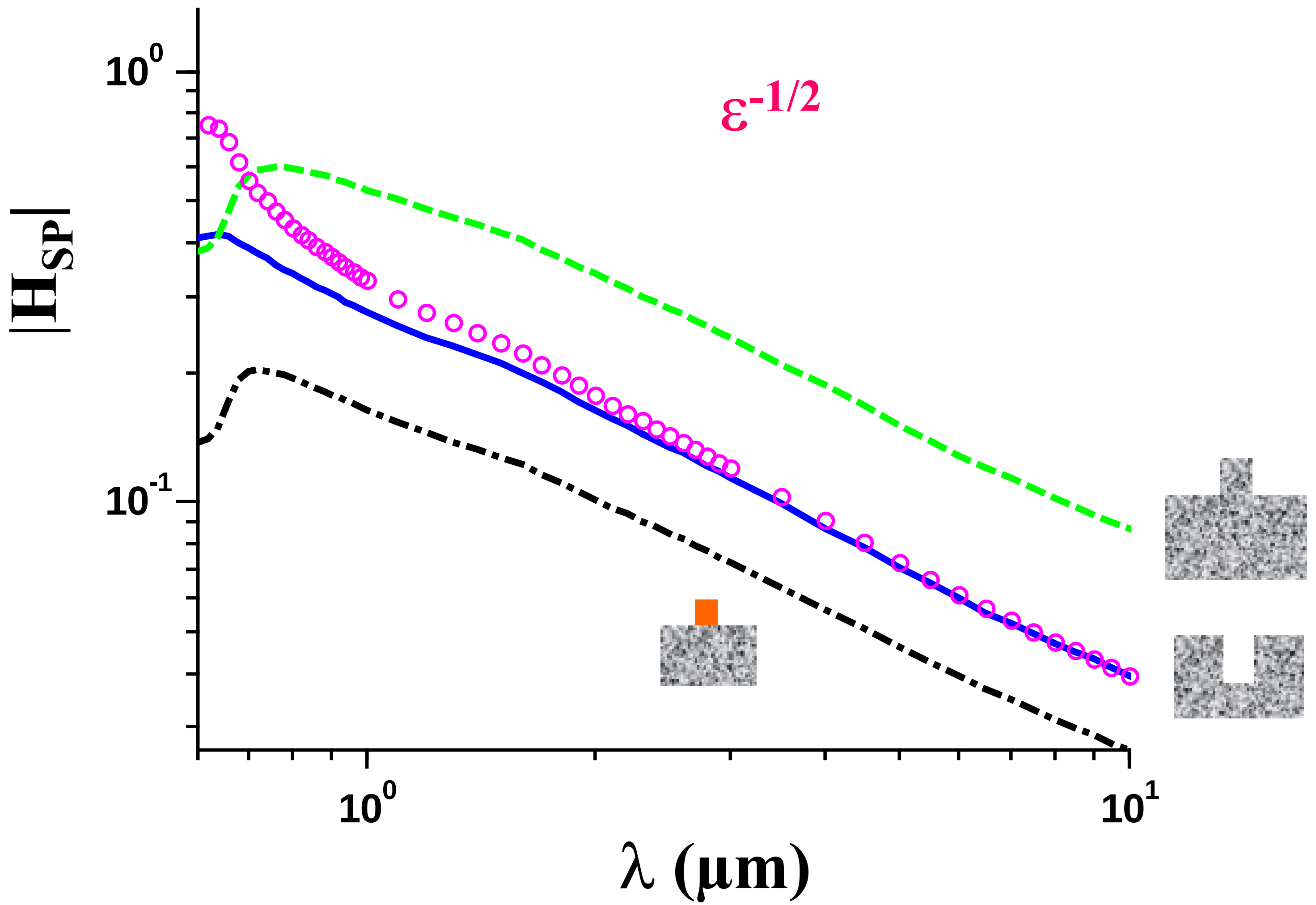

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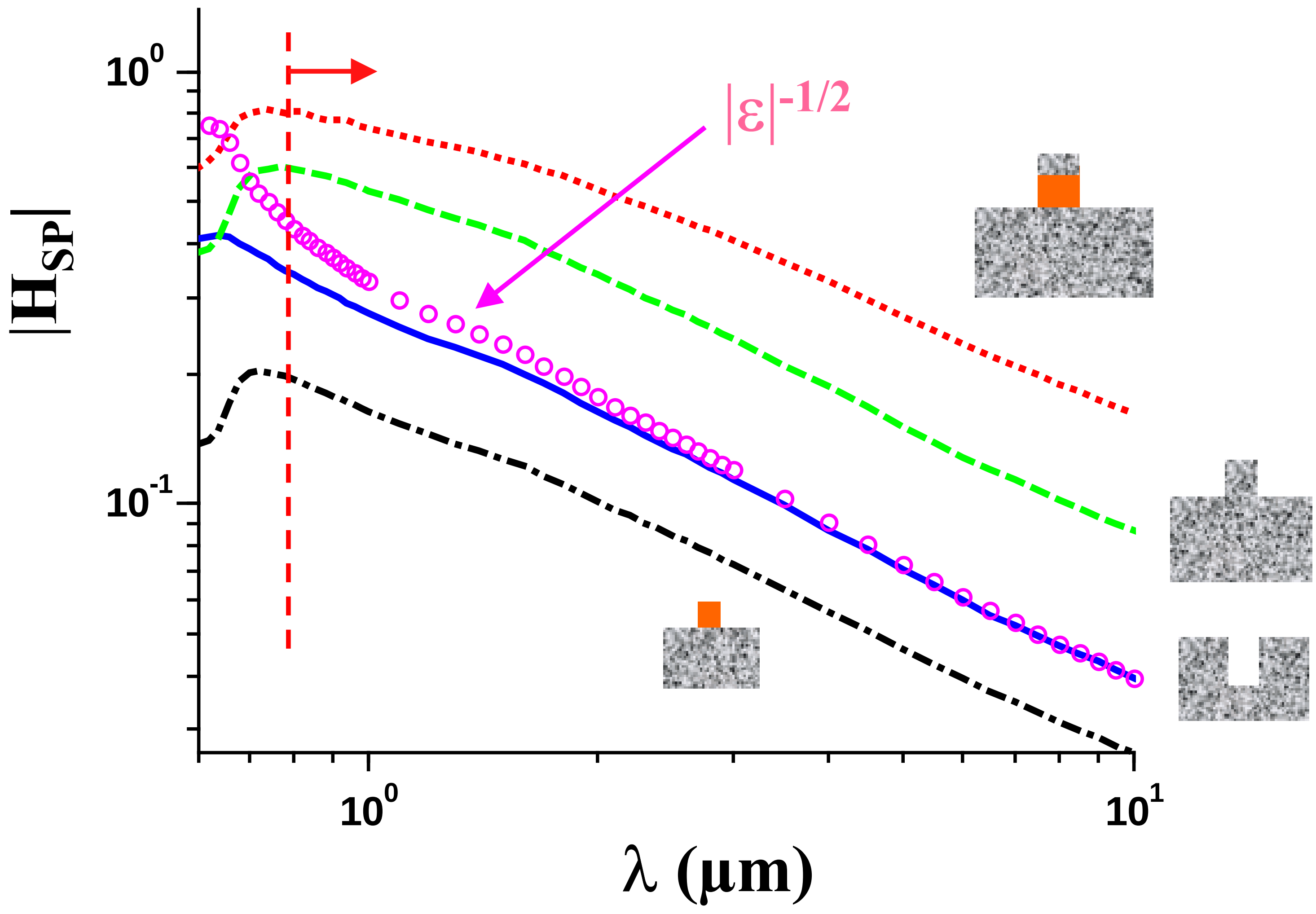


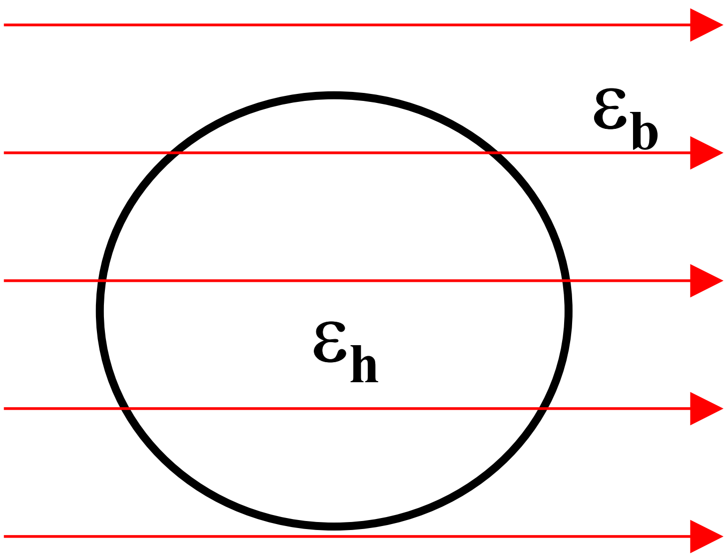
H. Liu et al., IEEE JSTQE 14, 1522 (2009 special issue)

(results obtained for gold)

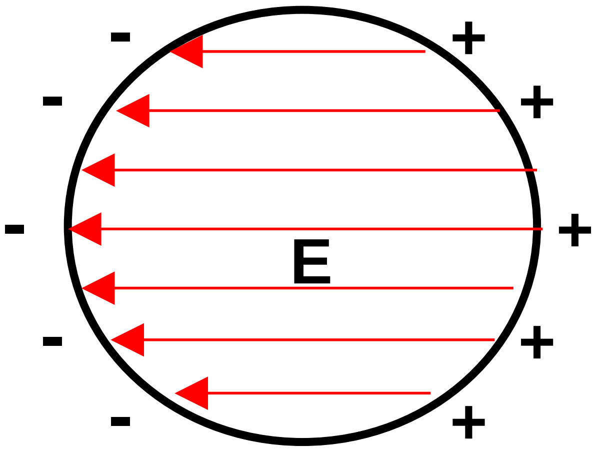
Local field corrections: small sphere ($R<<\lambda$)

$\Delta\varepsilon = \varepsilon_h - \varepsilon_b$

J.D. Jackson, *Classical Electrodynamics*



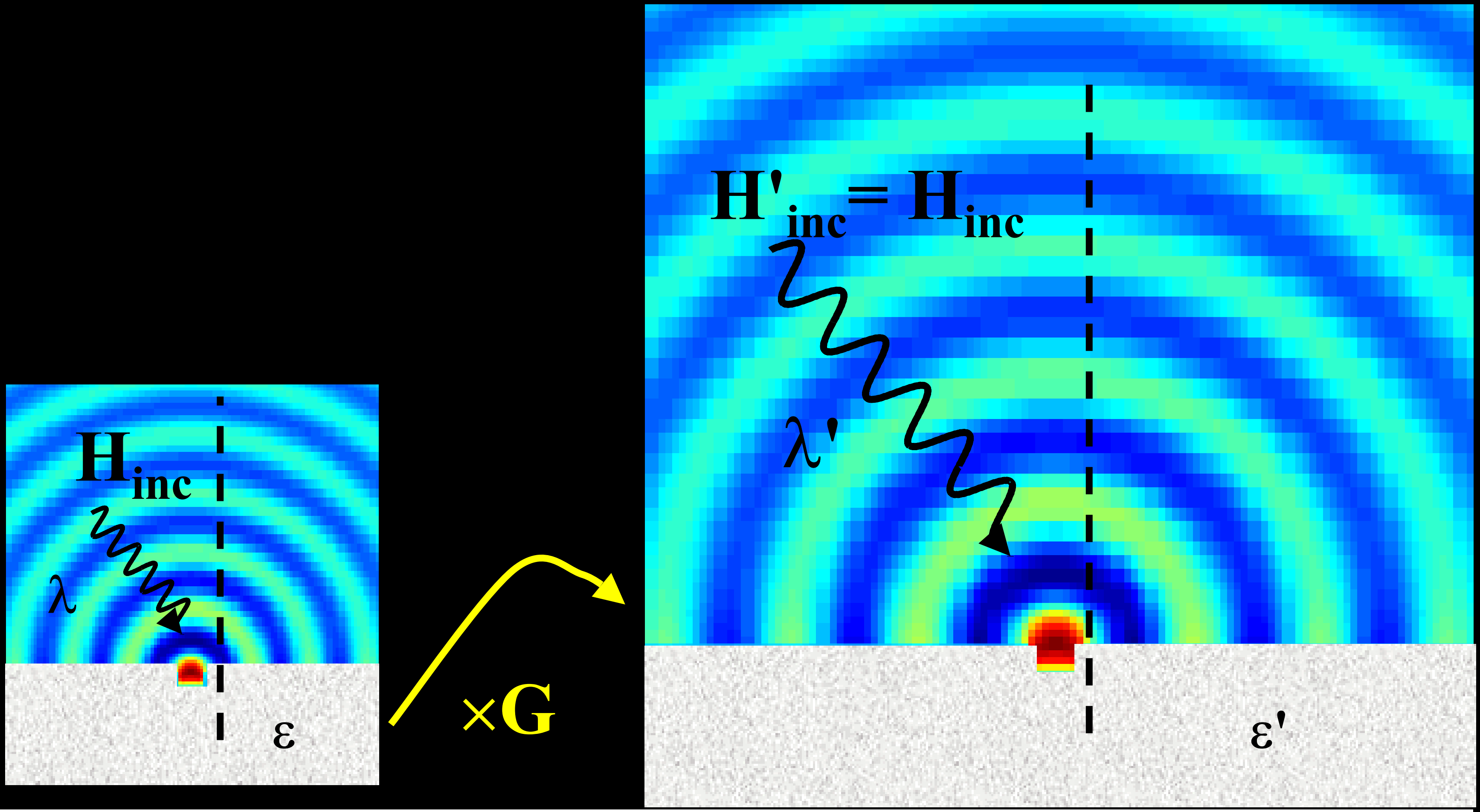
incident wave E_0



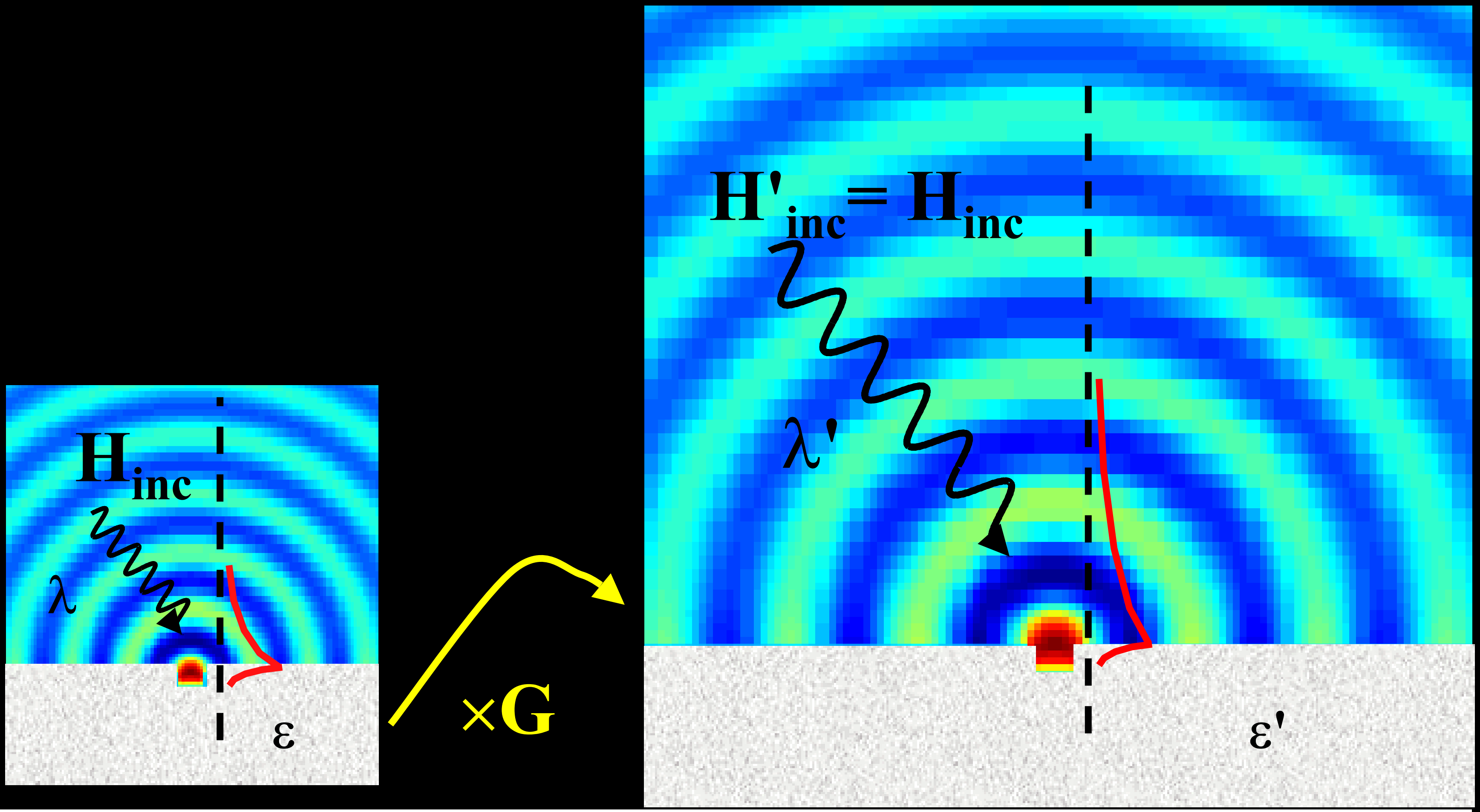
$$E = \frac{3\varepsilon_b}{\Delta\varepsilon + 3\varepsilon_b} E_0$$

$E = E_0$ for small $\Delta\varepsilon$ only !

Just plug $\Delta\varepsilon$ in perturbation formulas fails for large $\Delta\varepsilon$.



<u>assumptions</u> : the field distribution in the plane in the immediate vicinity of the slit becomes independent of the metallic permittivity (as $|\varepsilon|\to\infty$). (the particle dimensions are larger than the skin depth)



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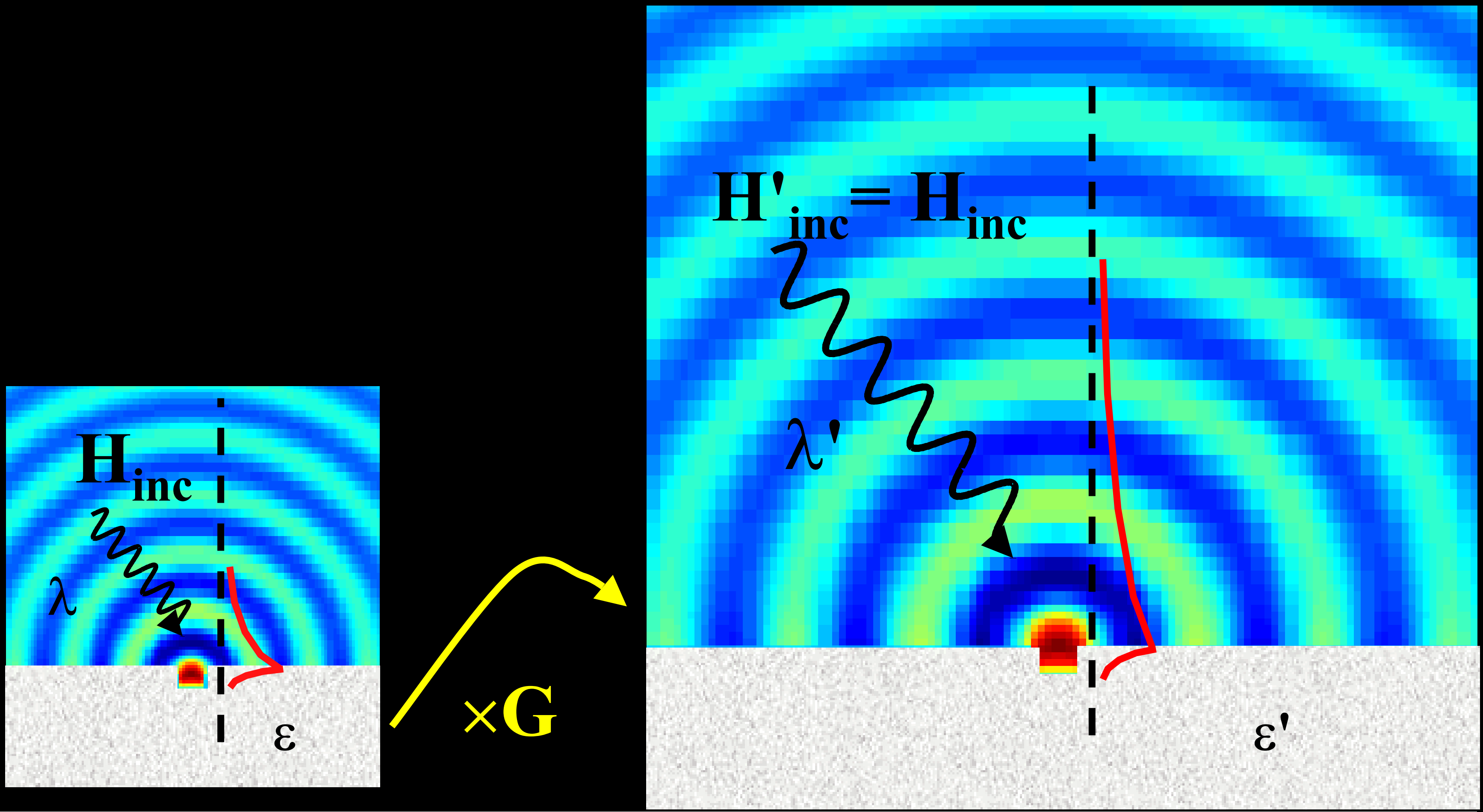

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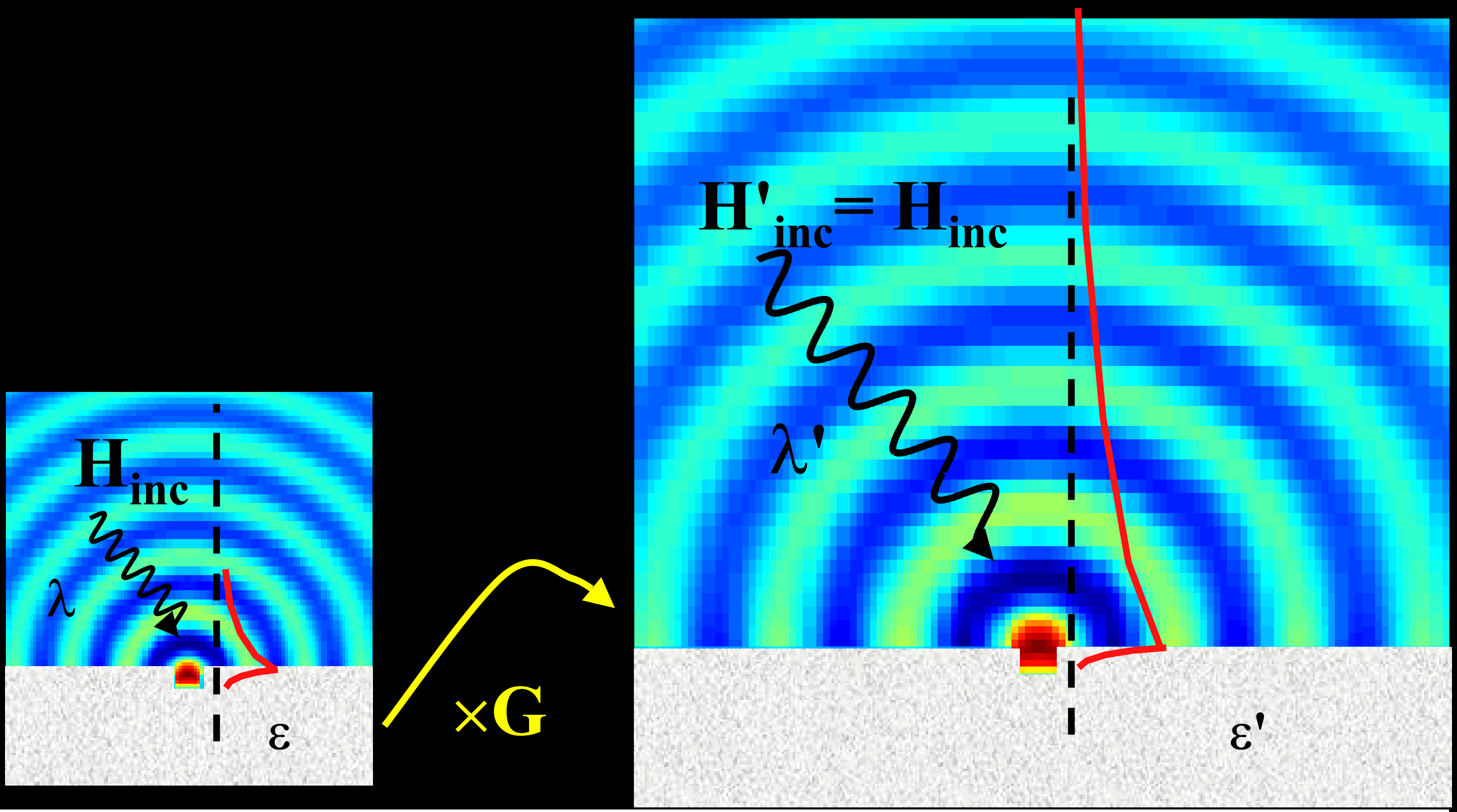

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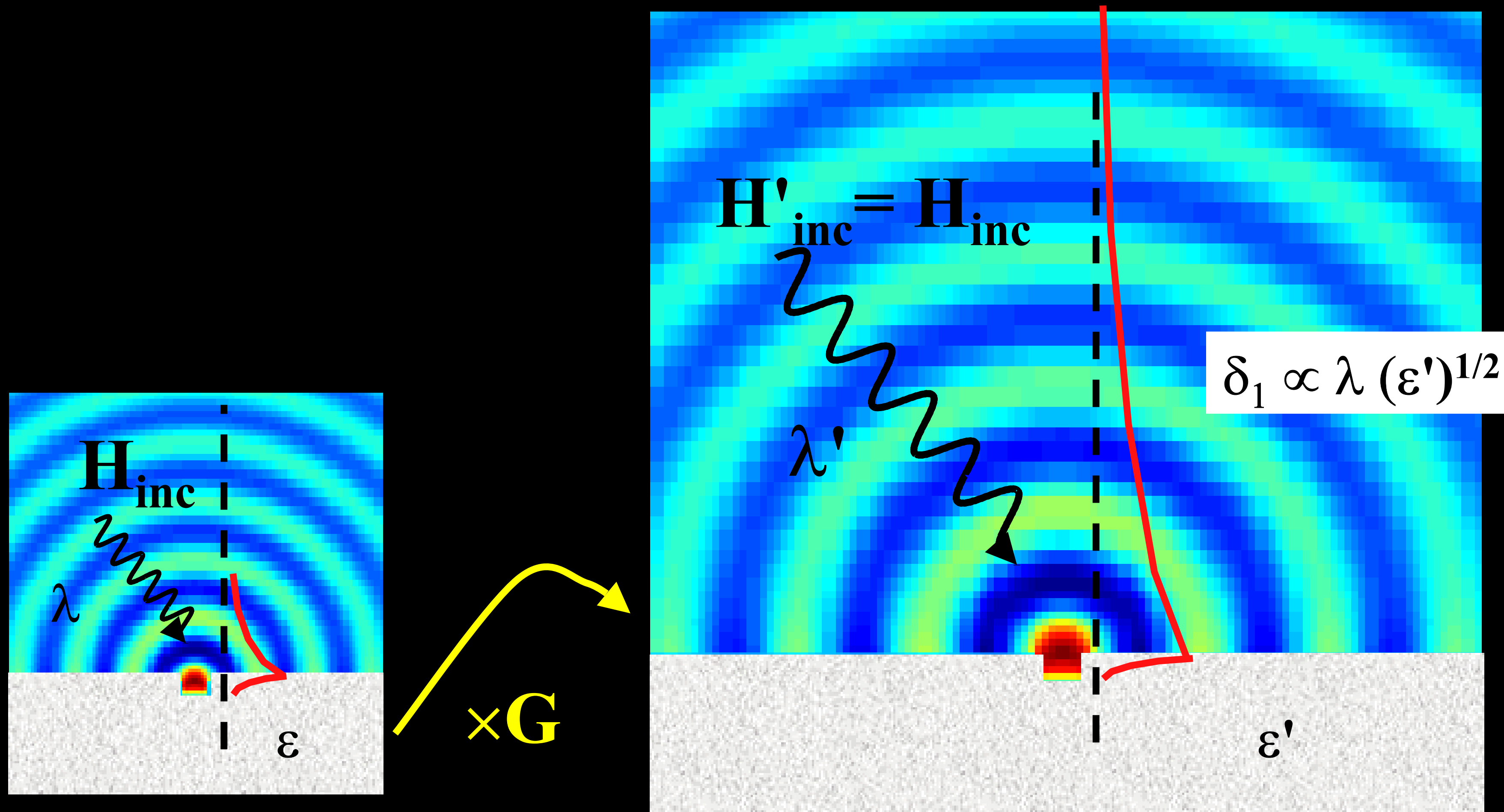

The initial launching of the SPP changes : $H_{SP} \propto |\varepsilon|^{-1/2}$

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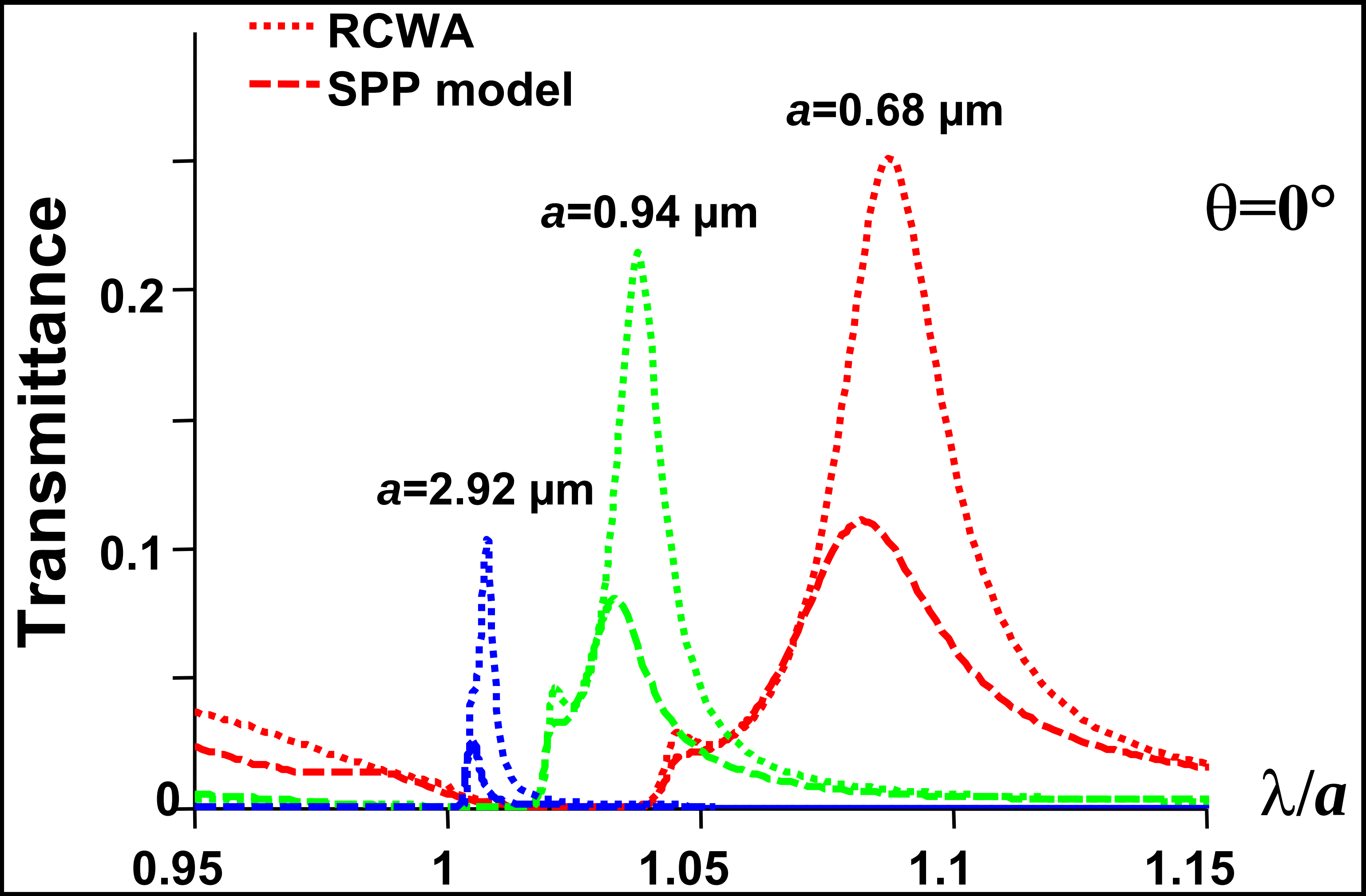
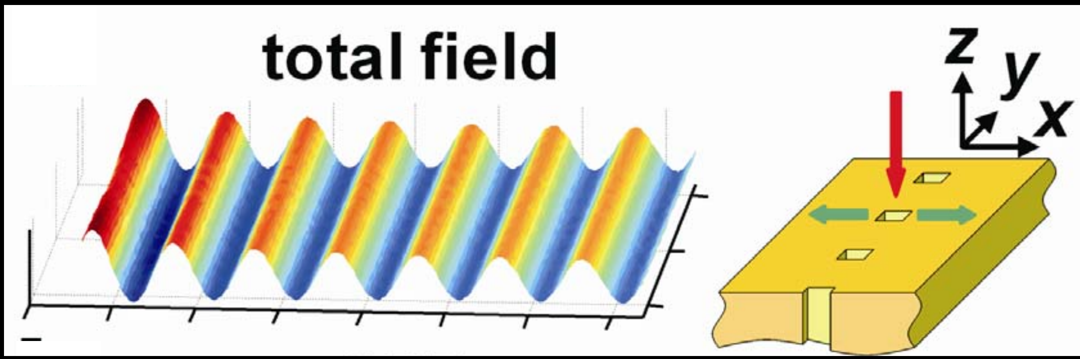
-definition of scattering coefficients for the quasi-CW
-dual wave picture microscopic model

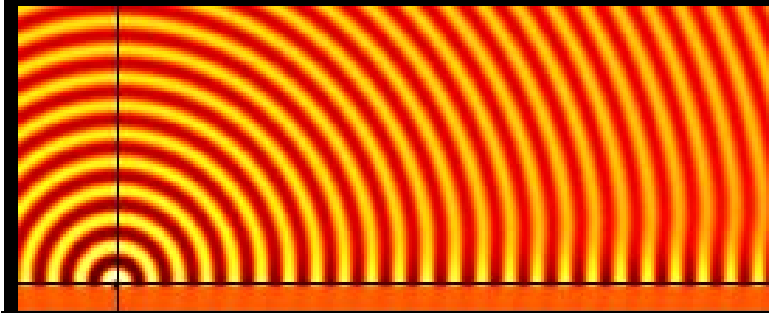
What is due to SPP in the EOT



Dual wave picture

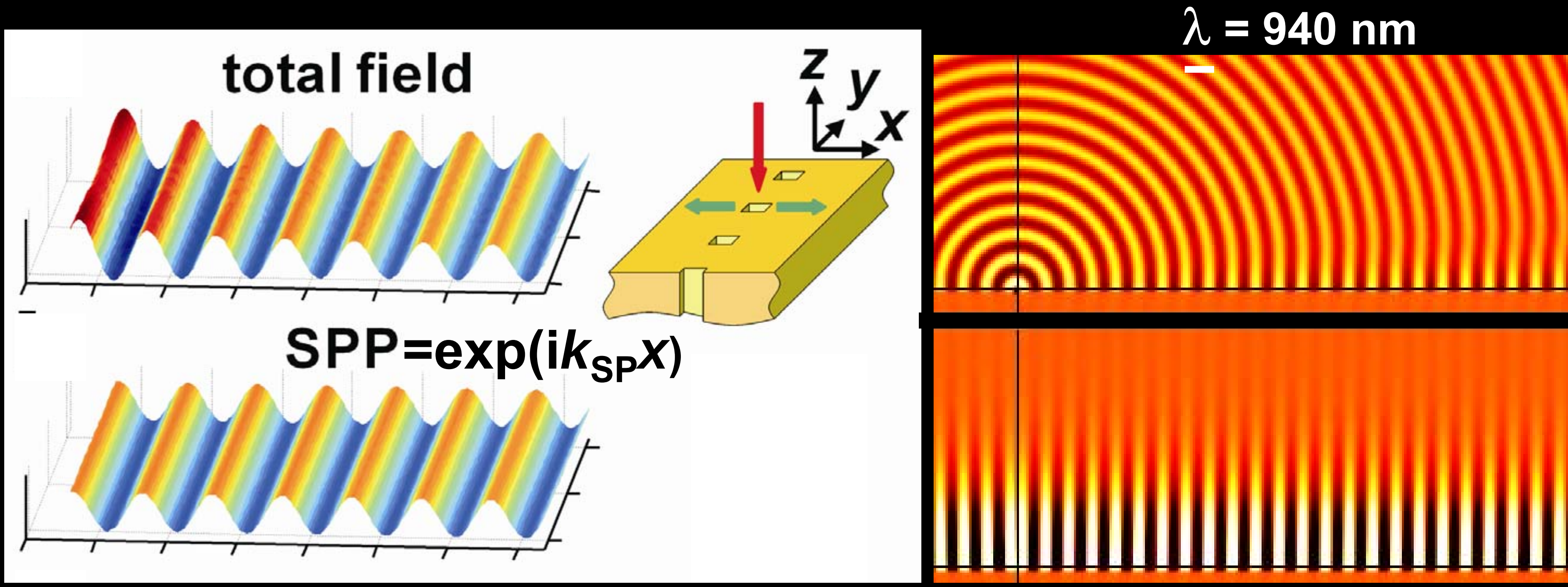


λ = 940 nm



P. Lalanne et J.P. Hugonin, Nature Phys. 2, 556 (2006).

Dual wave picture



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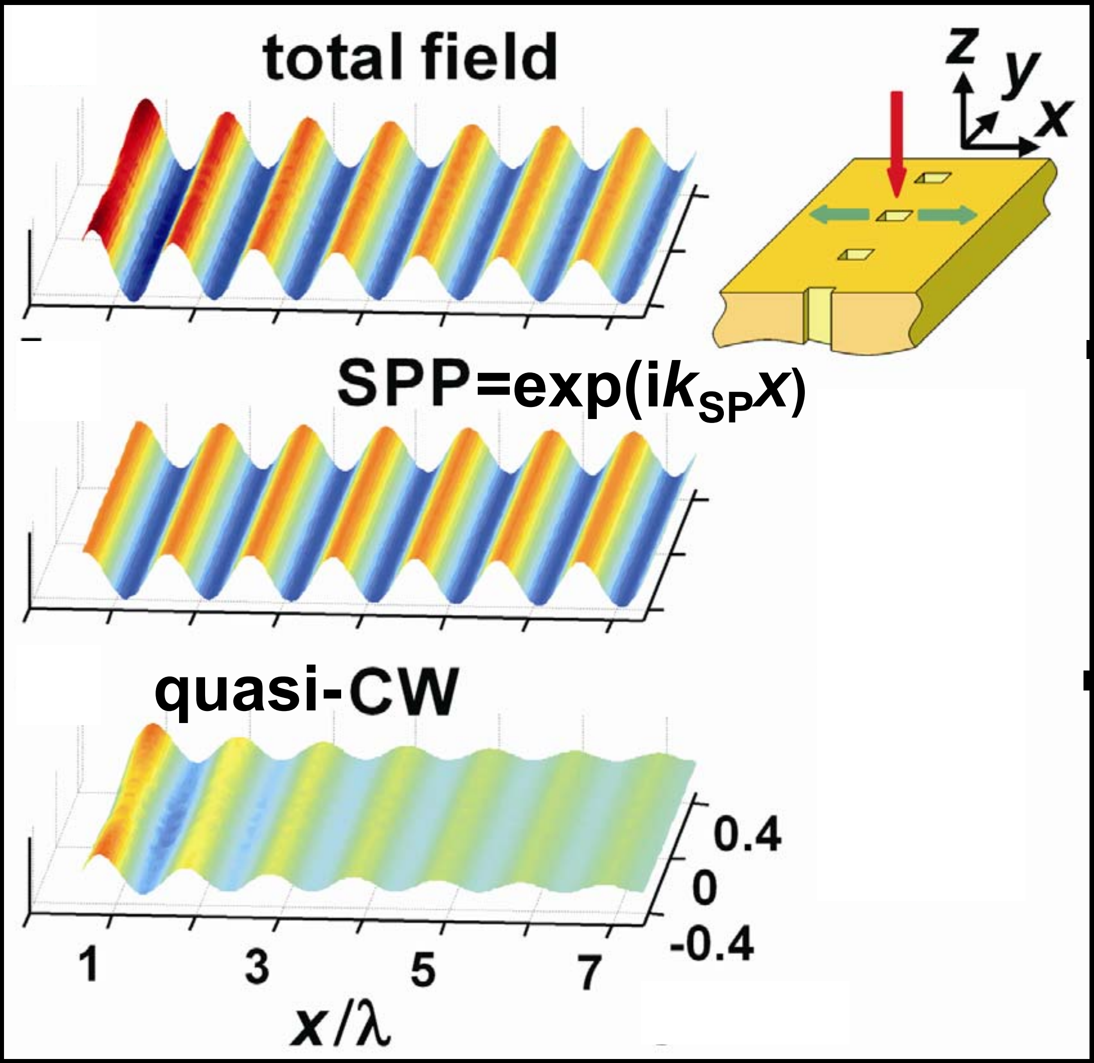
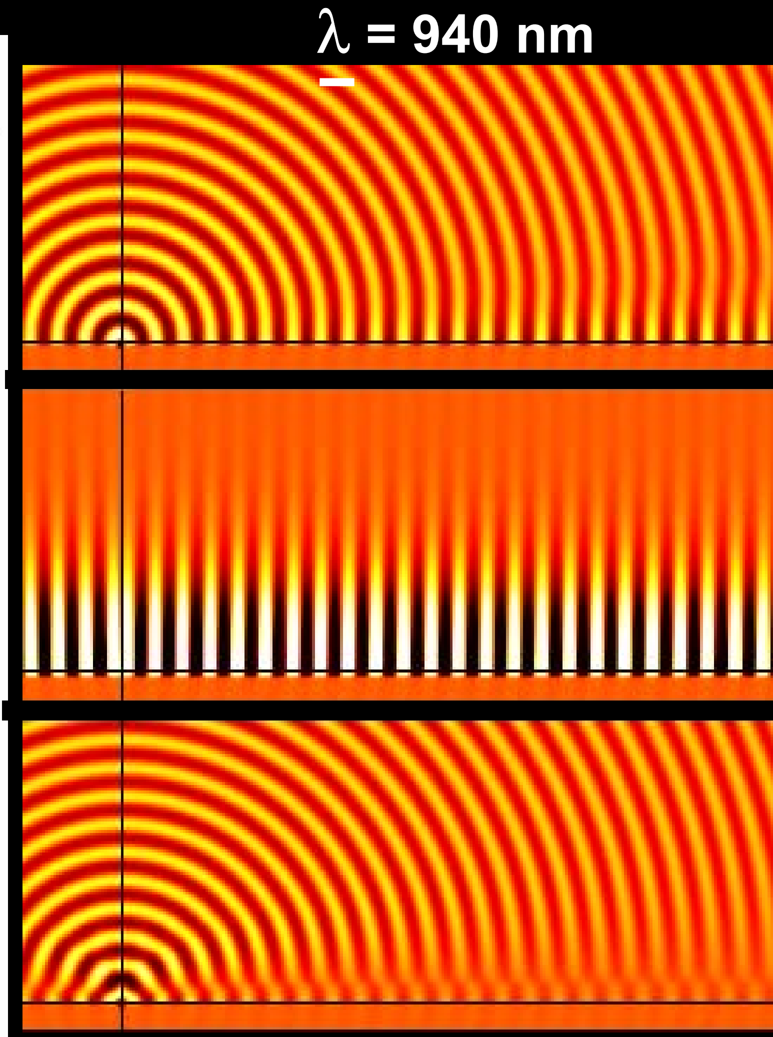
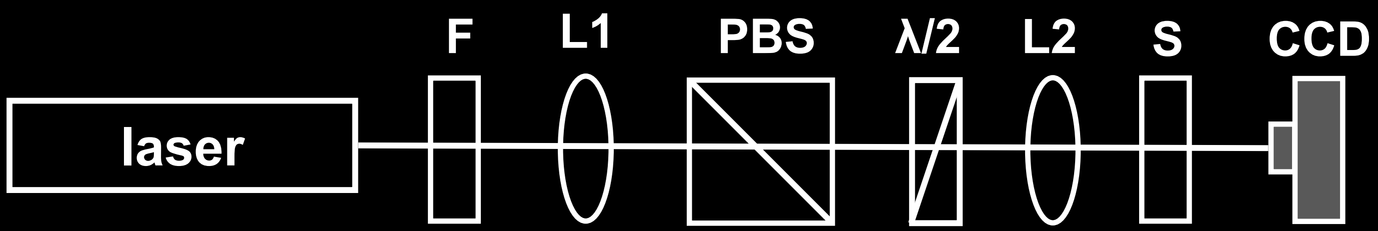


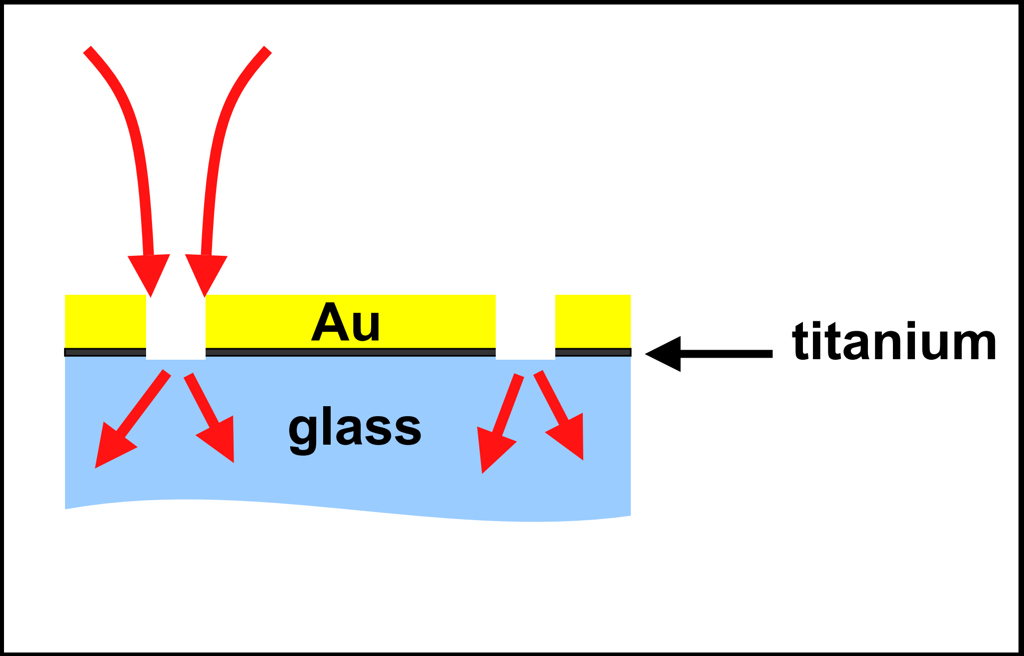


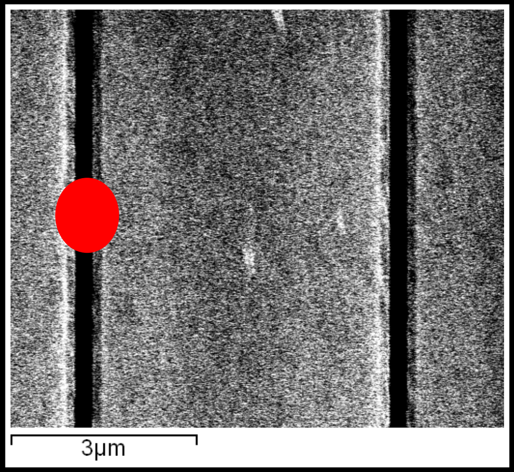
Why calling it a quasi-CW?

P. Lalanne et J.P. Hugonin, Nature Phys. 2, 556 (2006).

Young's slit experiment







N. Kuzmin et al., Opt. Lett. 32, 445 (2007).

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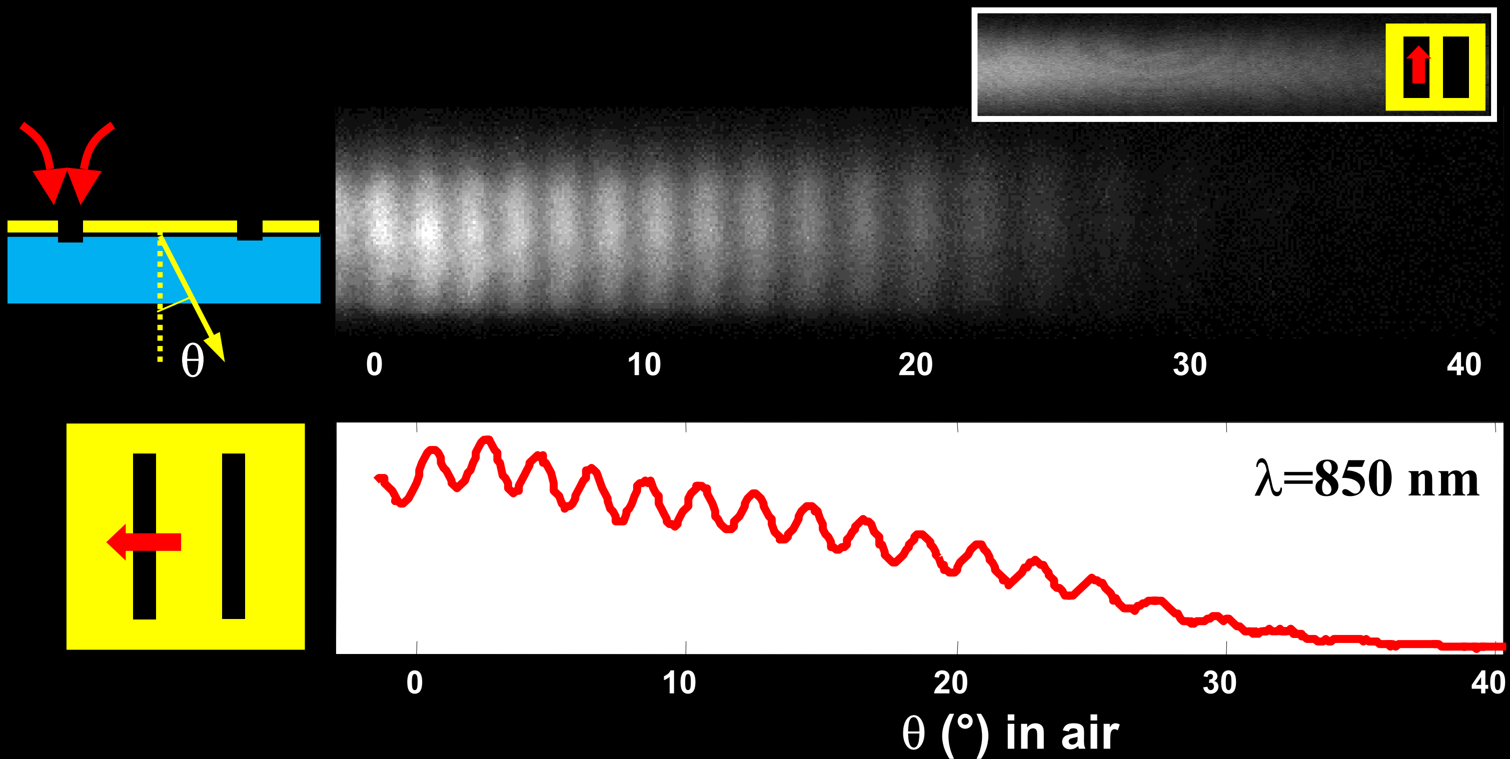
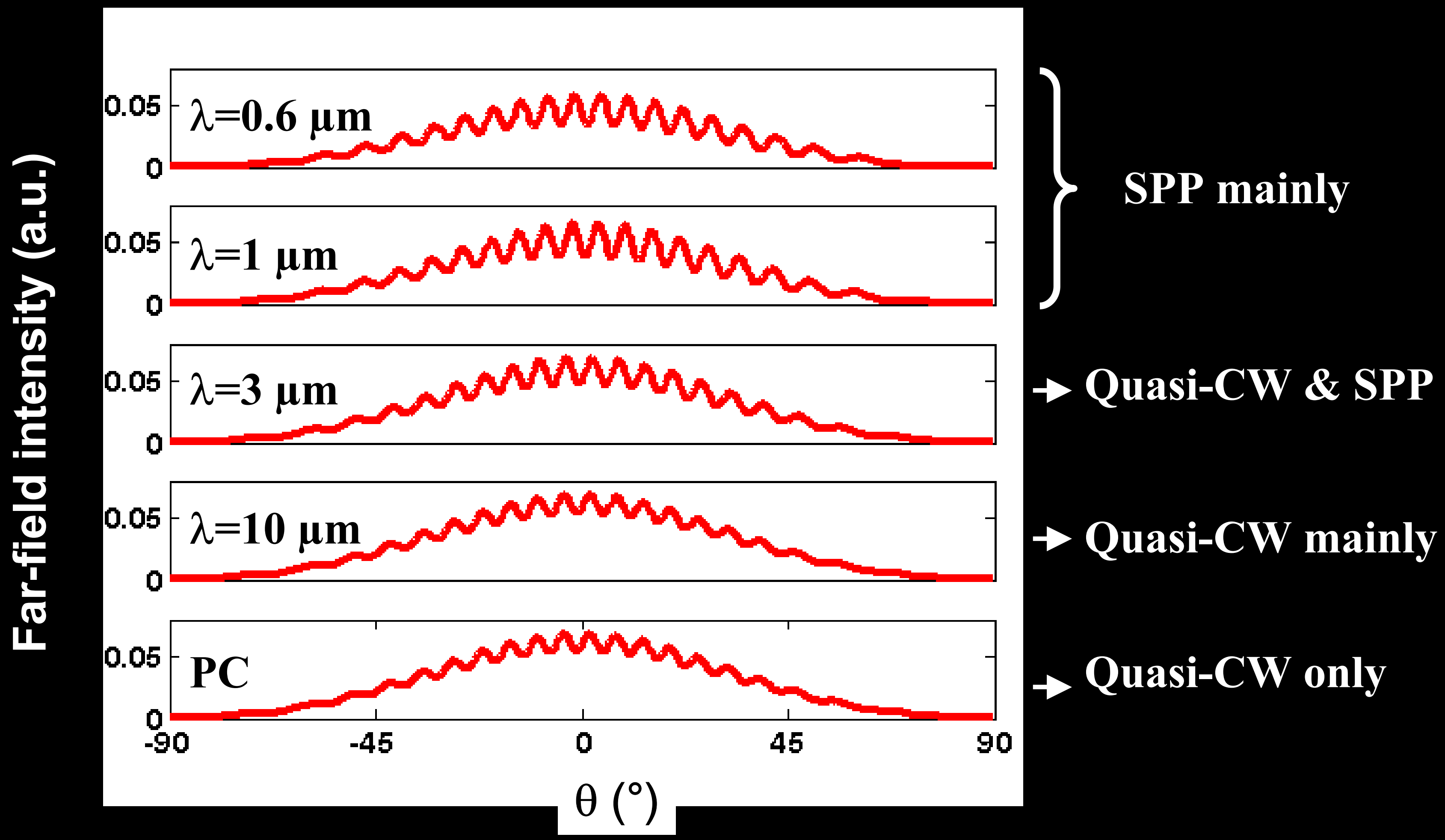


N. Kuzmin et al., Opt. Lett. 32, 445 (2007).
S. Ravets et al., JOSA B 26, B28 (2009).

Computational results



S. Ravets et al., JOSA B 26, B28 (2009).

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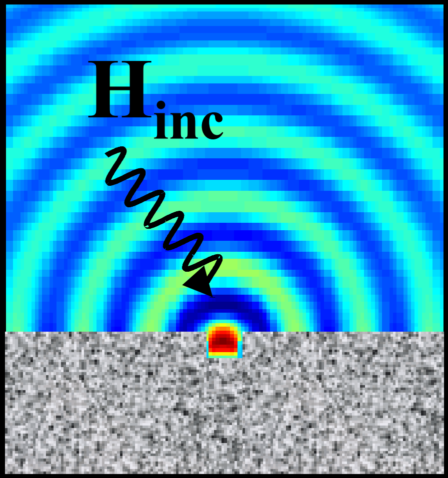


E_s, H_s= scattered field

E actual field

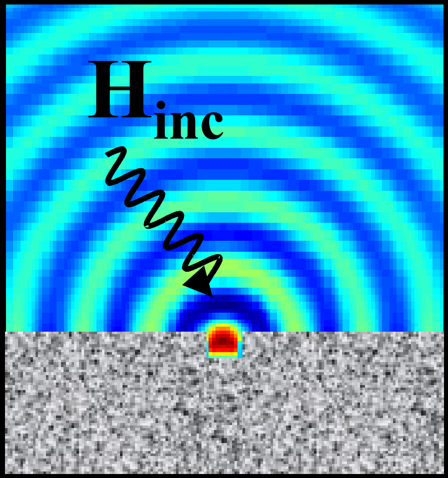
$$\begin{cases} \nabla\times E_s = j\omega\mu_0 H_s \\ \nabla\times H_s = -j\omega\varepsilon(r)E_s - \underline{j\omega\Delta\varepsilon E} \end{cases}$$

1/Hypothesis : the sub-λ indentation can be replaced by an effective dipole $p=p_x x+p_y y$. When i<u>s it reliable?</u>

Polarizability tensor

$p_x=(\alpha_{xx}E_{x,inc} + \alpha_{xz}E_{z,inc})x$

$p_z=(\alpha_{zx}E_{x,inc}+\alpha_{zz}E_{z,inc})z$



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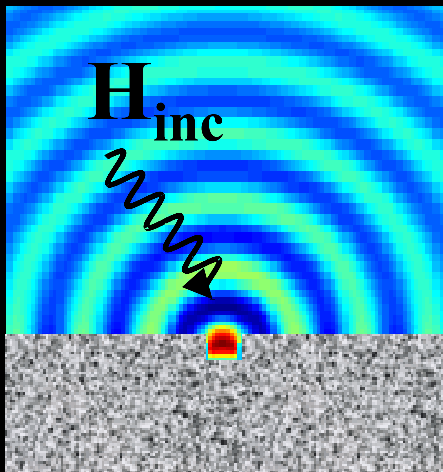
1/Hypothesis : the sub-λ indentation can be replaced by an effective dipole $p=p_x x+p_y y$.

2/The effective dipoles p_x and p_y are unknown. They probably depend on many parameters especially for sub-λ indentation that are not much smaller than λ (such as resonant grooves)



E_s, H_s = scattered field

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$$\begin{cases} \nabla\times E_s = j\omega\mu_0 H_s \\ \nabla\times H_s = -j\omega\varepsilon(r)E_s \underline{-j\omega\Delta\varepsilon E} \end{cases}$$

1/Hypothesis : the sub-λ indentation can be replaced by an effective dipole $p=p_x x+p_y y$.

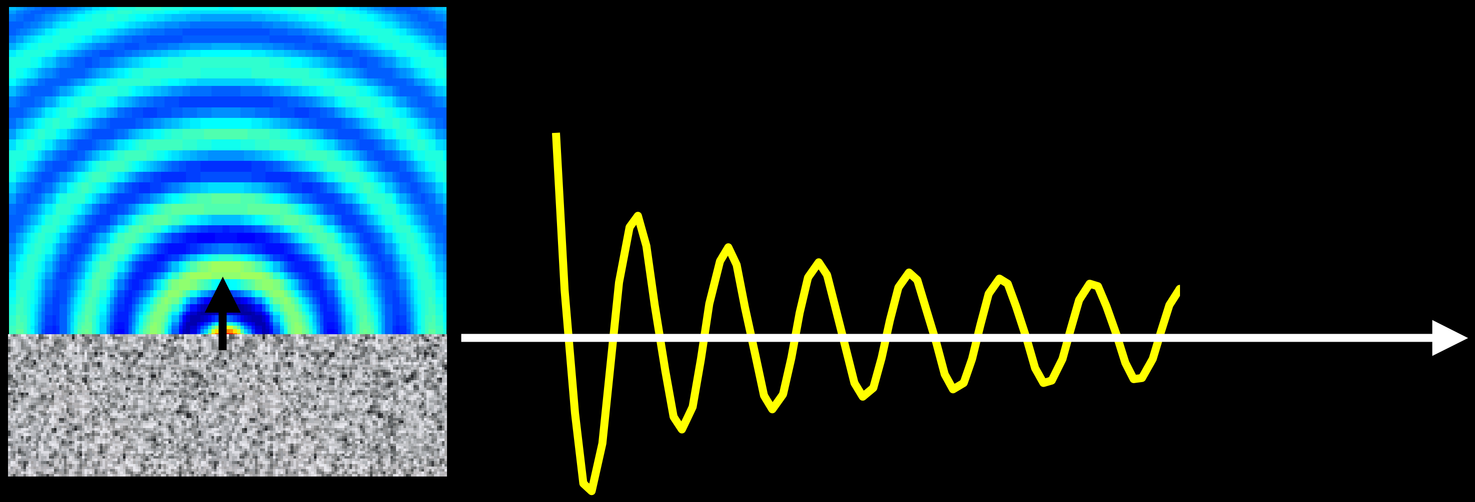
2/The effective dipoles p_x and p_y are unknown.

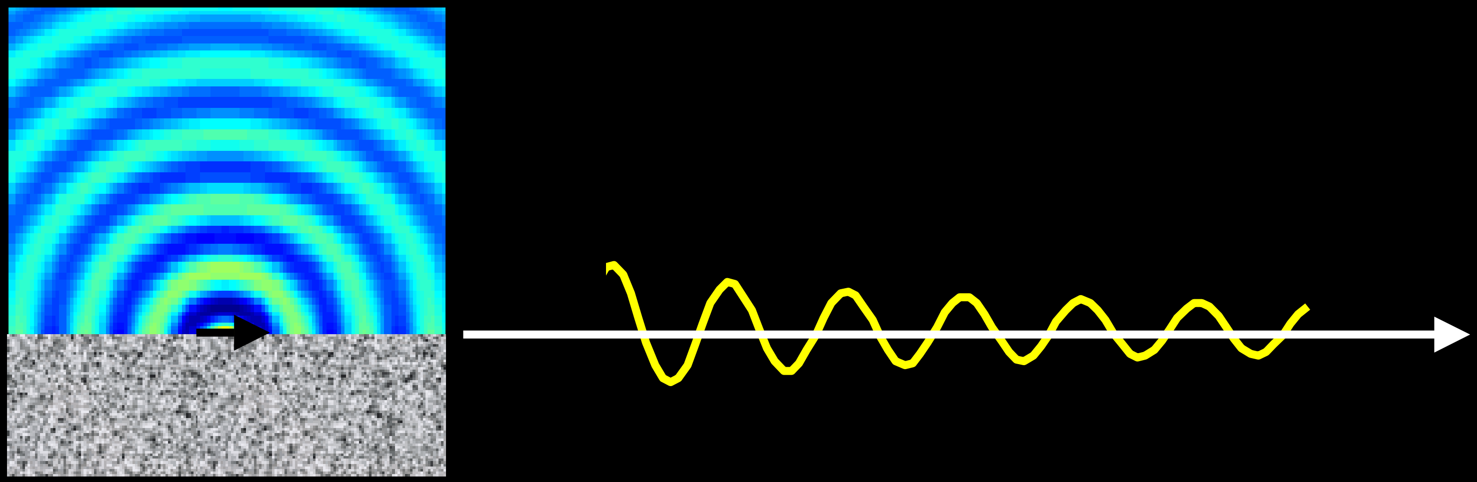
3/ We solve Maxwell's equation for both dipole source

$$\nabla\times E = j\omega\mu_0 H$$

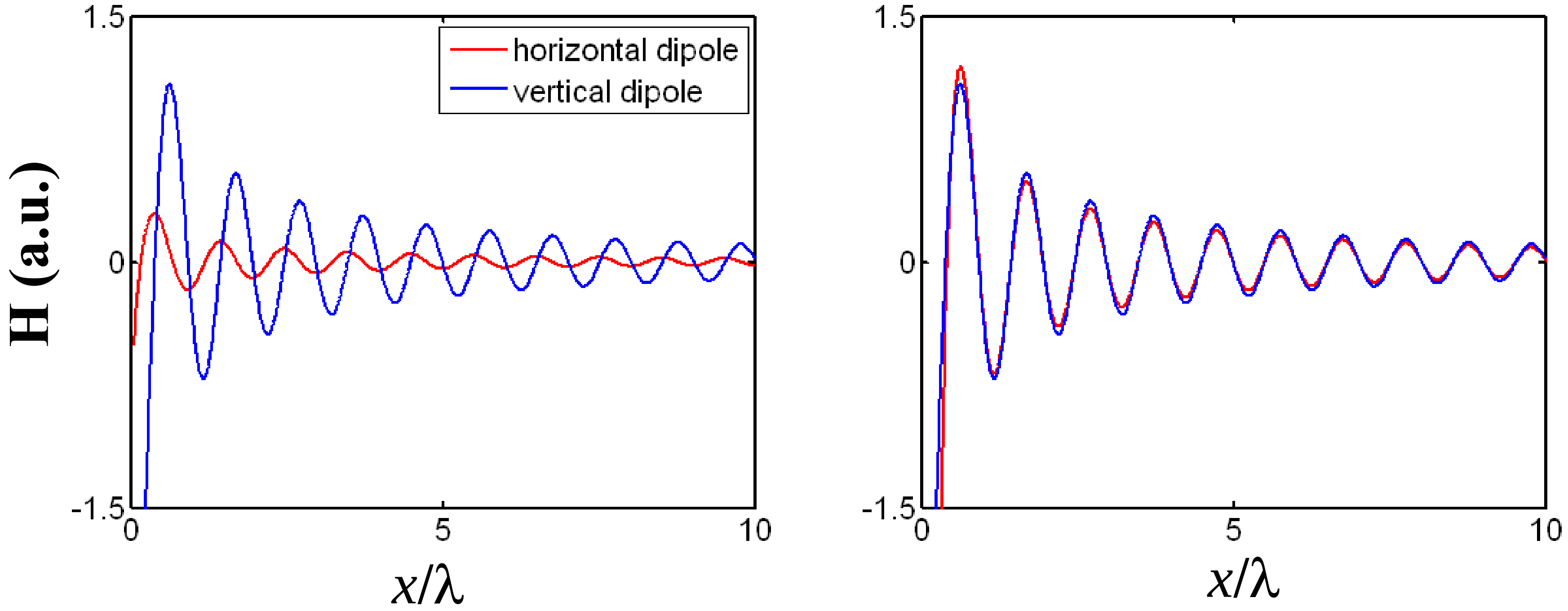
$$\nabla\times H = -j\omega\varepsilon(r)E + (E_{s,x}x + E_{s,y}y)\,\delta(x,y)$$

The bad scenario



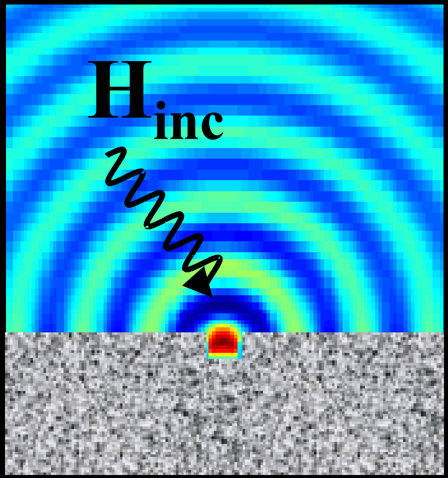


The actual scenario



λ=800 nm, gold

The two dipole sources approximately generate the same field



E_s, H_s= scattered field

E actual field

$$\begin{cases} \nabla\times E_s = j\omega\mu_0 H_s \\ \nabla\times H_s = -j\omega\varepsilon(r)E_s \underline{-j\omega\Delta\varepsilon E} \end{cases}$$

1/Hypothesis : the sub-λ indentation can be replaced by an effective dipole $p=p_x x+p_y y$.

2/The effective dipoles p_x and p_y are unknown.

3/ The field scattered (in the vicinity of the surface for a given frequency) has always the same shape :

$$\Phi(x,y)= [\alpha_{SP}E_{inc}]\ \Phi_{SP}(x,y) + [\alpha_{CW}E_{inc}]\ \Phi_{CW}(x,y)$$

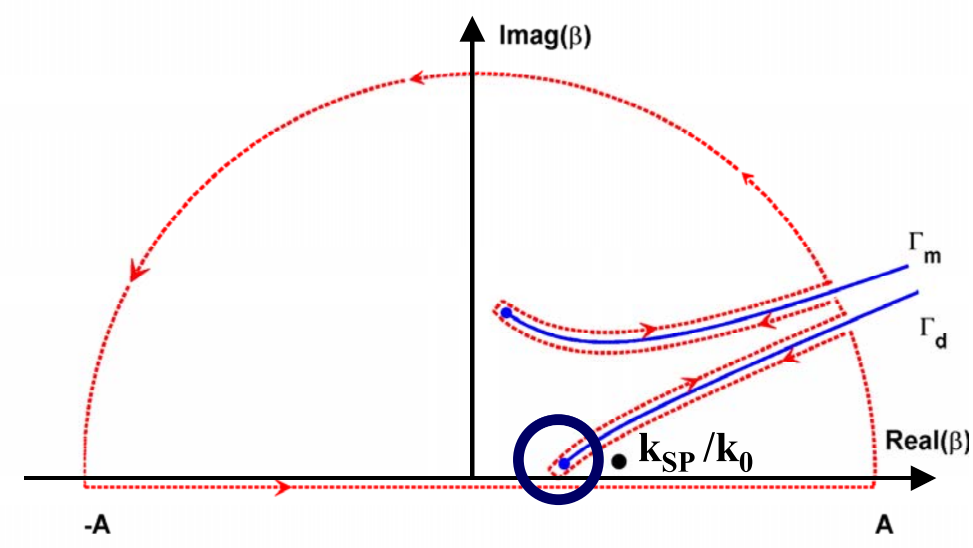
Analytical expression for the quasi-CW

Cauchy theorem

$$H(x,z=0)=\int_{-\infty}^{\infty}\frac{\exp(ik_0\beta x)d\beta}{i\left(\sqrt{\varepsilon_d-\beta^2}/\varepsilon_d+\sqrt{\varepsilon_m-\beta^2}/\varepsilon_m\right)}$$

$$\mathbf{H = H_{SP} + H_{CW}}$$

$$\mathbf{H_{SP}}=\frac{k_{SP}^2}{k_0^2}\frac{\sqrt{\varepsilon_d\varepsilon_m}}{\varepsilon_m-\varepsilon_d}\exp(ik_{SP}x)$$



H_{CW}= Integral over a single real variable

Dominated by the branch-point singularity

A single pole singularity : SPP contribution

Two Branch-cut singularities : Γ_m and Γ_d

Maths have been initially developped for the transmission theory in wireless telegraphy with a Hertzian dipole radiating over the earth

I. Zenneck, Propagation of plane electromagnetic waves along a plane conducting surface and its bearing on the theory of transmission in wireless telegraphy, Ann. Phys (1907) 23, 846–866.

R.W.P. King and M.F. Brown, Lateral electromagnetic waves along plane boundaries: a summarizing approach, Proc. IEEE (1984) 72, 595–611.

R. E. Collin, Hertzian dipole radiating over a lossy earth or sea: some early and late 20th-century controversies, IEEE Antennas Propag. Mag. (2004) 46, 64–79.

0/H. J. Lezec and T. Thio, "Diffracted evanescent wave model for enhanced and suppressed optical transmission through subwavelength hole arrays", Opt. Exp. 12, 3629-41 (2004).
1/G. Gay, O. Alloschery, B. Viaris de Lesegno, C. O'Dwyer, J. Weiner, H. J. Lezec, "The optical response of nanostructured surfaces and the composite diffracted evanescent wave model", Nature Phys. 2, 262-267 (2006).
2/PL and J.P. Hugonin, Nature Phys. 2, 556 (2006).
3/B. Ung, Y.L. Sheng, "Optical surface waves over metallo-dielectric nanostructures", Opt. Express (2008) 16, 9073–9086.
4/Y. Ravel, Y.L. Sheng, "Rigorous formalism for the transient surface Plasmon polariton launched by subwavelength slit scattering", Opt. Express (2008) 16, 21903–21913.
5/W. Dai & C. Soukoulis, "Theoretical analysis of the surface wave along a metal-dielectric interface", PRB accepted for publication (private communication).
6/L. Martin Moreno, F. Garcia-Vidal, SPP4 proceedings 2009.
7/PL, J.P. Hugonin, H. Liu and B. Wang, "A microscopic view of the electromagnetic properties of sub-λ metallic surfaces", Surf. Sci. Rep. (review article under proof corrections, see ArXiv too)

Analytical expression for the quasi-CW

Maxwell's equations

$$\nabla \times E = j\omega\mu_0 H$$

$$\nabla \times H = -j\omega\varepsilon(r)E + (E_{s,x}x + E_{s,y}y)\,\delta(x,y)$$

Analytical solution

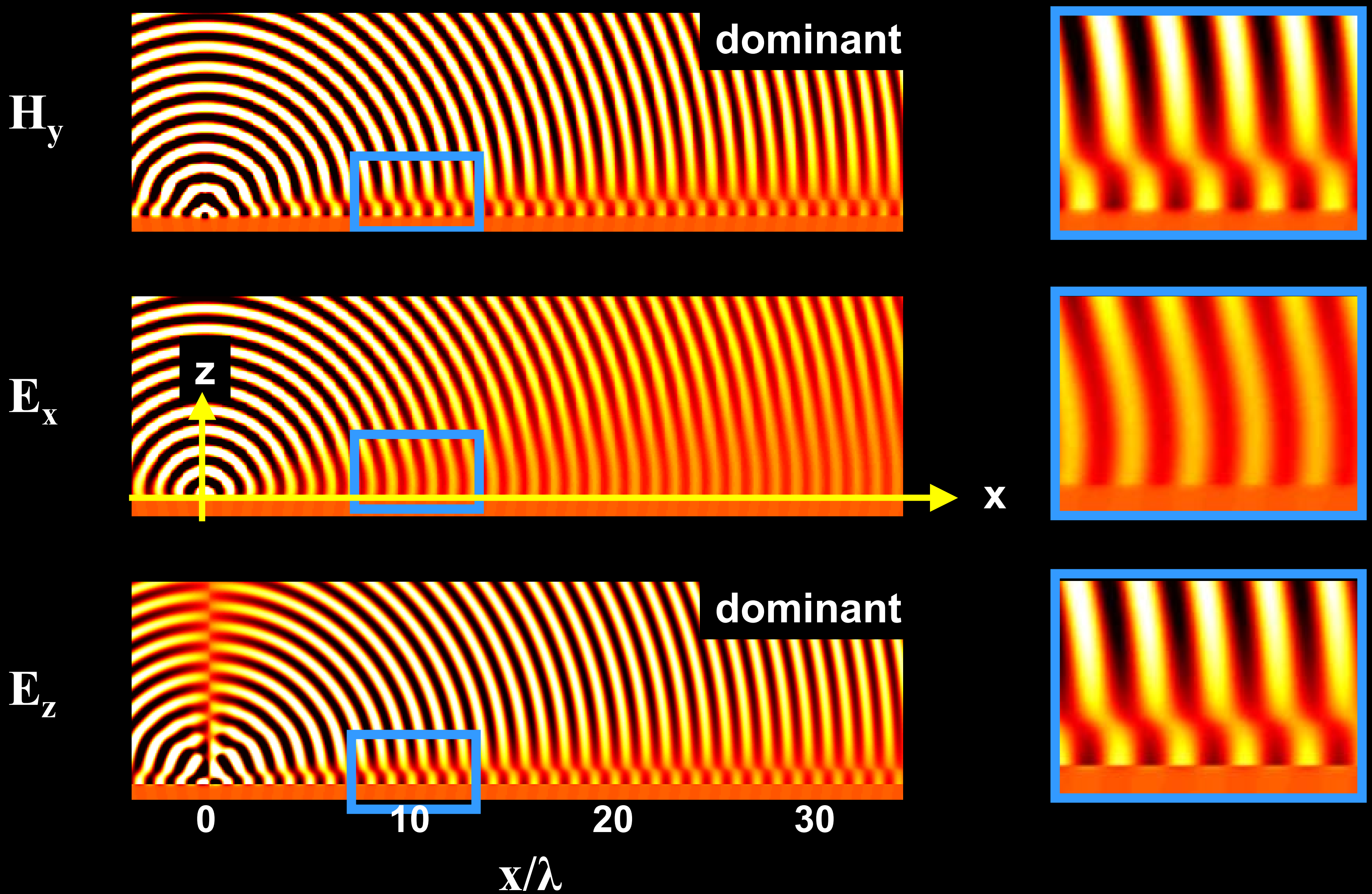
$$[\chi_m/\varepsilon_m E_{s,x} + n_d/\varepsilon^S E_{s,y}] \times \begin{pmatrix} H_{z,CW} \\ E_{x,CW} \\ E_{y,CW} \end{pmatrix}$$

ε_S is either ε_d or ε_m, whether the Dirac source is located in the dielectric material or in the metallic medium

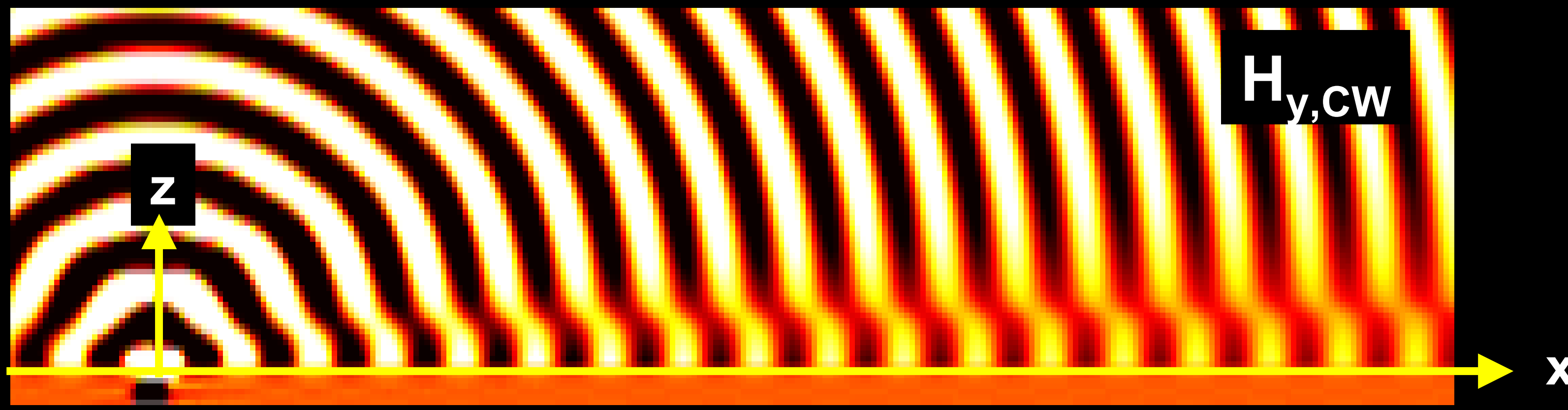
Under the Hypothesis that

- $|\varepsilon_m| >> \varepsilon_d$
- $z < \lambda$
- $x > \lambda/2\pi$

quasi-CW for gold at λ=940 nm



Intrinsic properties of quasi-CW



Hypothesis
- $z < \lambda$
- $x > \lambda/2\pi$

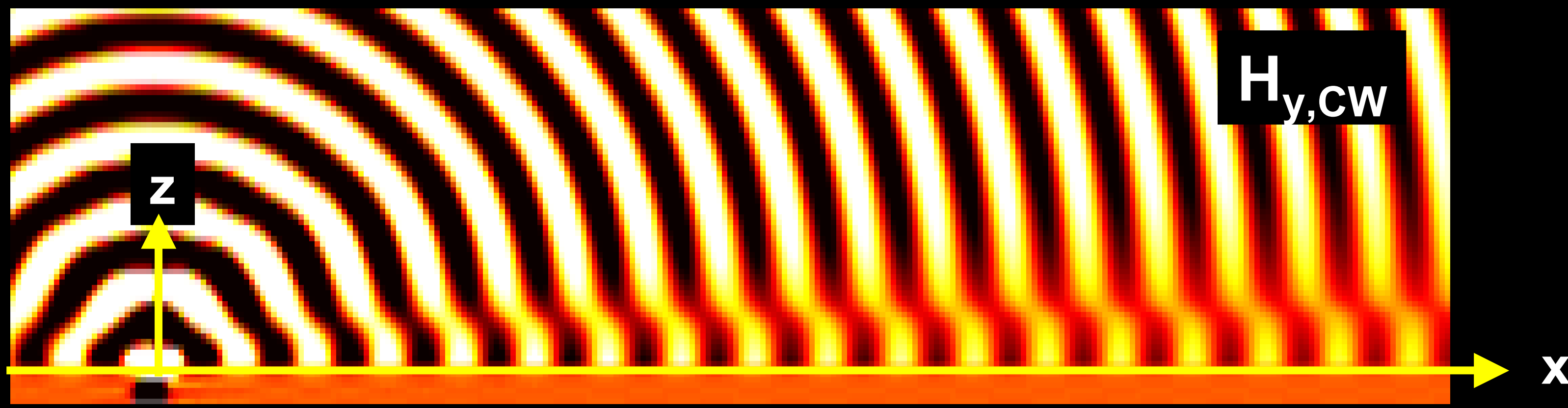
$$\rightarrow \begin{pmatrix} H_{y,CW} \\ E_{x,CW} \\ E_{z,CW} \end{pmatrix} = F(x) \begin{pmatrix} H_{y,0} \\ E_{x,0} \\ E_{z,0} \end{pmatrix}$$

- **F(x) is a slowly-varying envelop**
- **$[H_{y,0}, E_{x,0}, E_{z,0}]$ is the normalized field associated to the limit case of the reflection of a plane-wave at grazing incidence**

PL et al., Surf. Sci. Rep. (review article under production, 2009)

Grazing plane-wave field

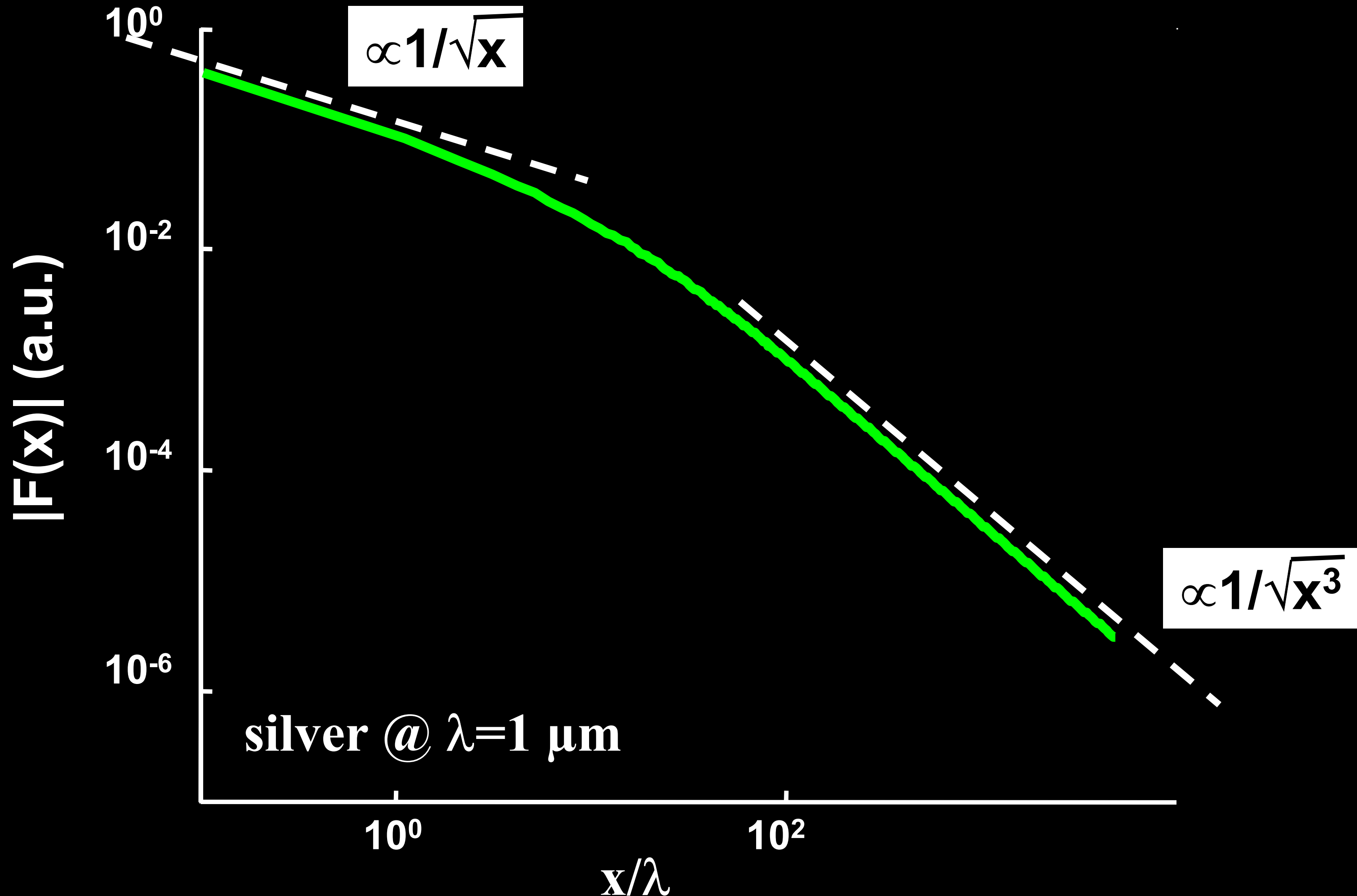
$$\begin{pmatrix} H_{y,CW} \\ E_{x,CW} \\ E_{z,CW} \end{pmatrix} = F(x) \begin{pmatrix} H_{y,0} \\ E_{x,0} \\ E_{z,0} \end{pmatrix}$$



- linear z-dependence for $z < \lambda$
- Main fields are almost null for $z \approx (\lambda/2\pi)\,|\varepsilon_m|^{1/2}$
- nearly an $\exp(ik_0x)$ x-dependence for $x >> \lambda$

PL et al., Surf. Sci. Rep. (review article under production, 2009)

Closed-form expression for F(x)



PL et al., Surf. Sci. Rep. (review article under production, 2009)

Closed-form expression for F(x)

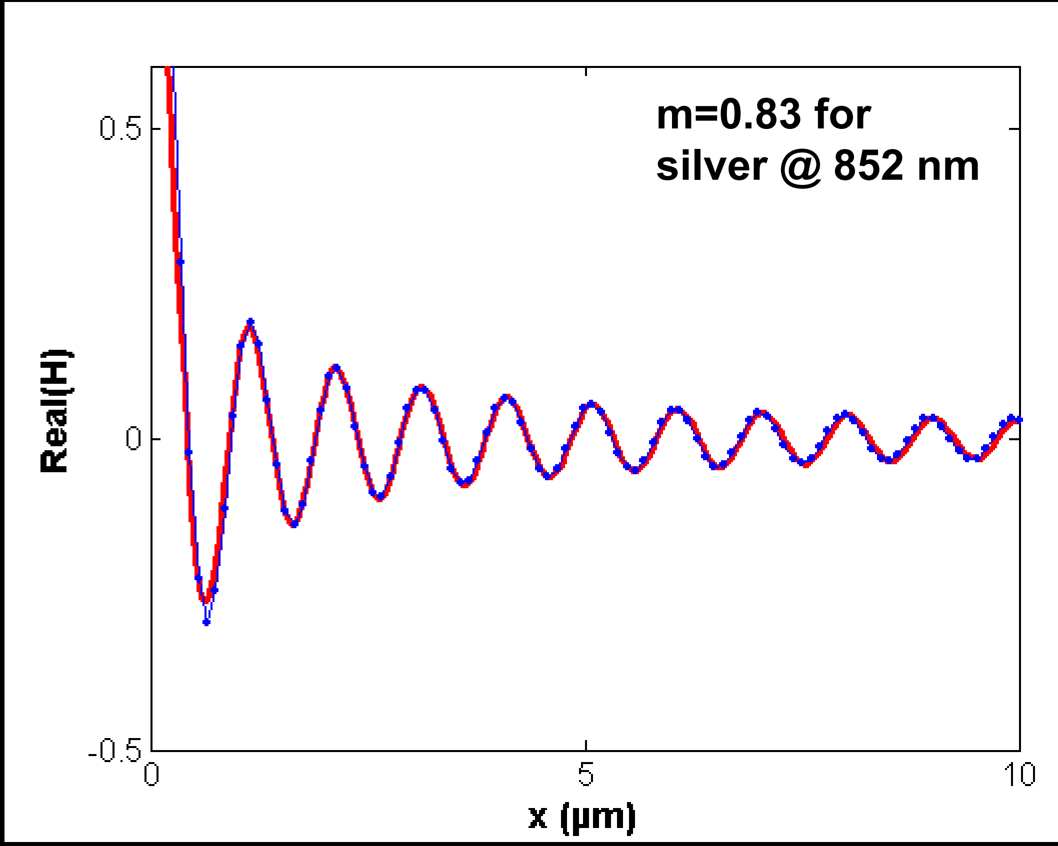
Highly accurate form for any *x*

$F(x) = \exp(ik_0x)\, W[2\pi(n_{SP}-n_d)x/\lambda]\, (x/\lambda)^{3/2}$ with W(t) an Erf–like function

Highly accurate for x < 10λ

$F(x) = \exp(ik_0x)\, (x/\lambda)^{-m}$

m varies from 0.9 in the visible to 0.5 in the far IR



1. The emblematic example of the EOT

- extraordinary optical transmission (EOT)
- limitation of classical "macroscopic" grating theories
- a microscopic pure-SPP model of the EOT

2. SPP generation by 1D sub-λ indentation

- rigorous calculation (orthogonality relationship)
- the important example of slit
- scaling law with the wavelength

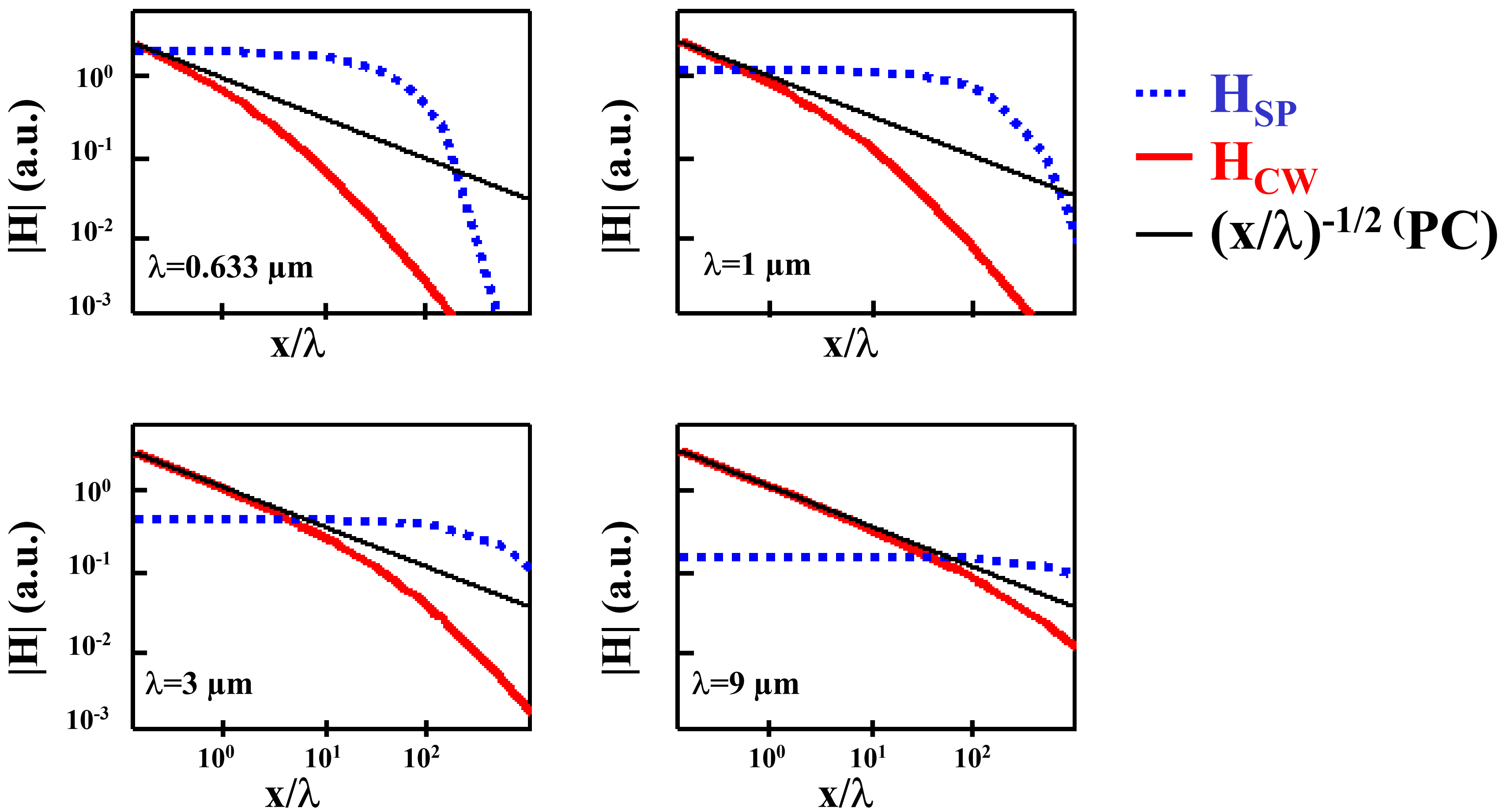
3. The quasi-cylindrical wave

- the importance of the quasi-CW
- definition & properties
- scaling law with the wavelength

4. Microscopic theory of sub-λ surfaces

- definition of scattering coefficients for the quasi-CW
- dual wave picture microscopic model

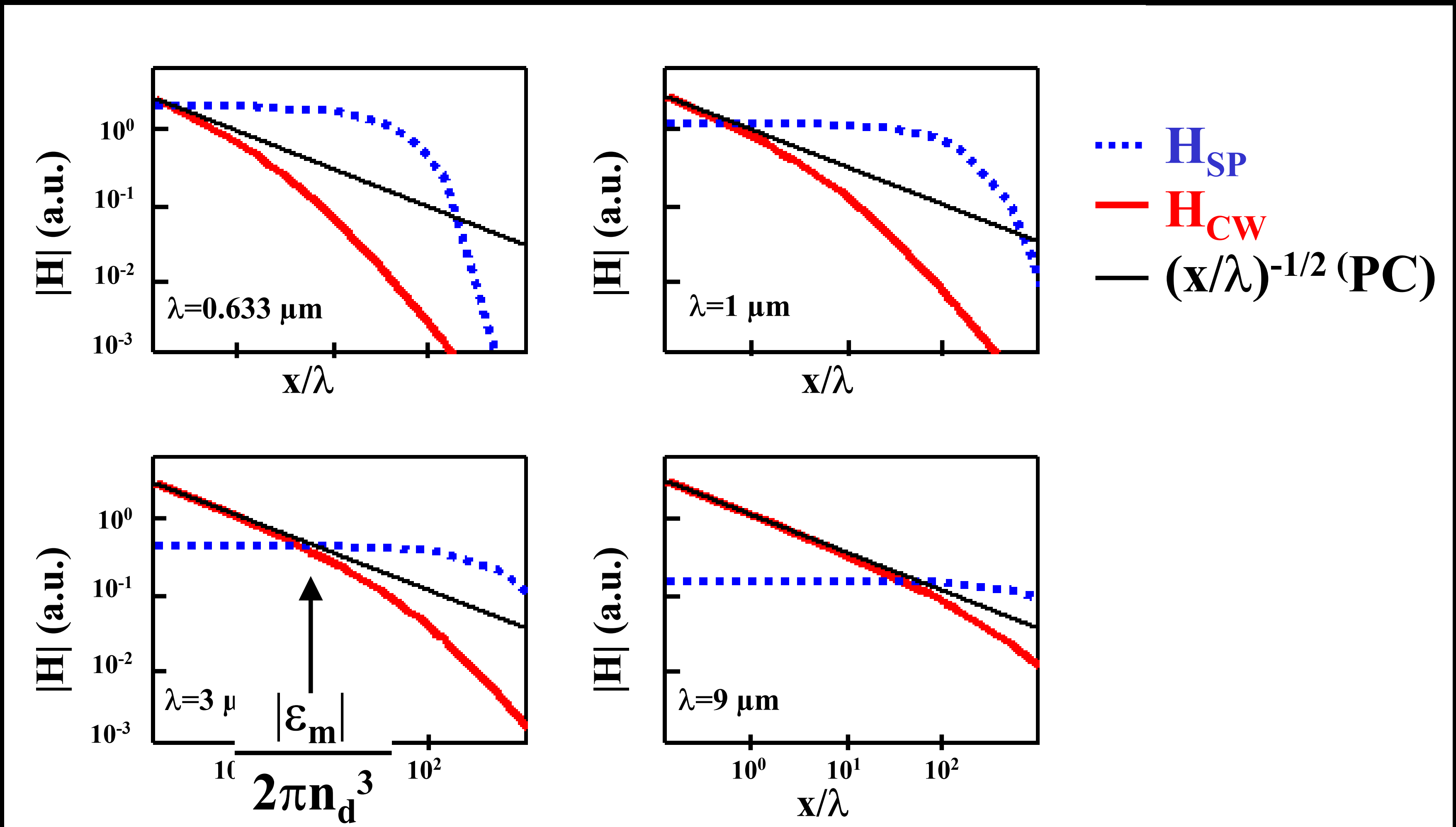
Scaling law



(result for silver)

PL and J.P. Hugonin, Nature Phys. 2, 556 (2006)

Scaling law



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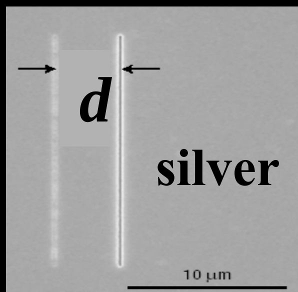
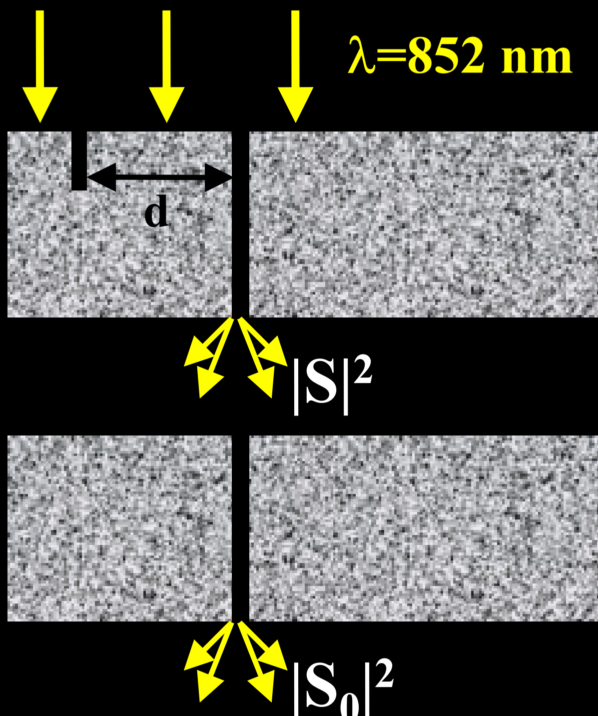
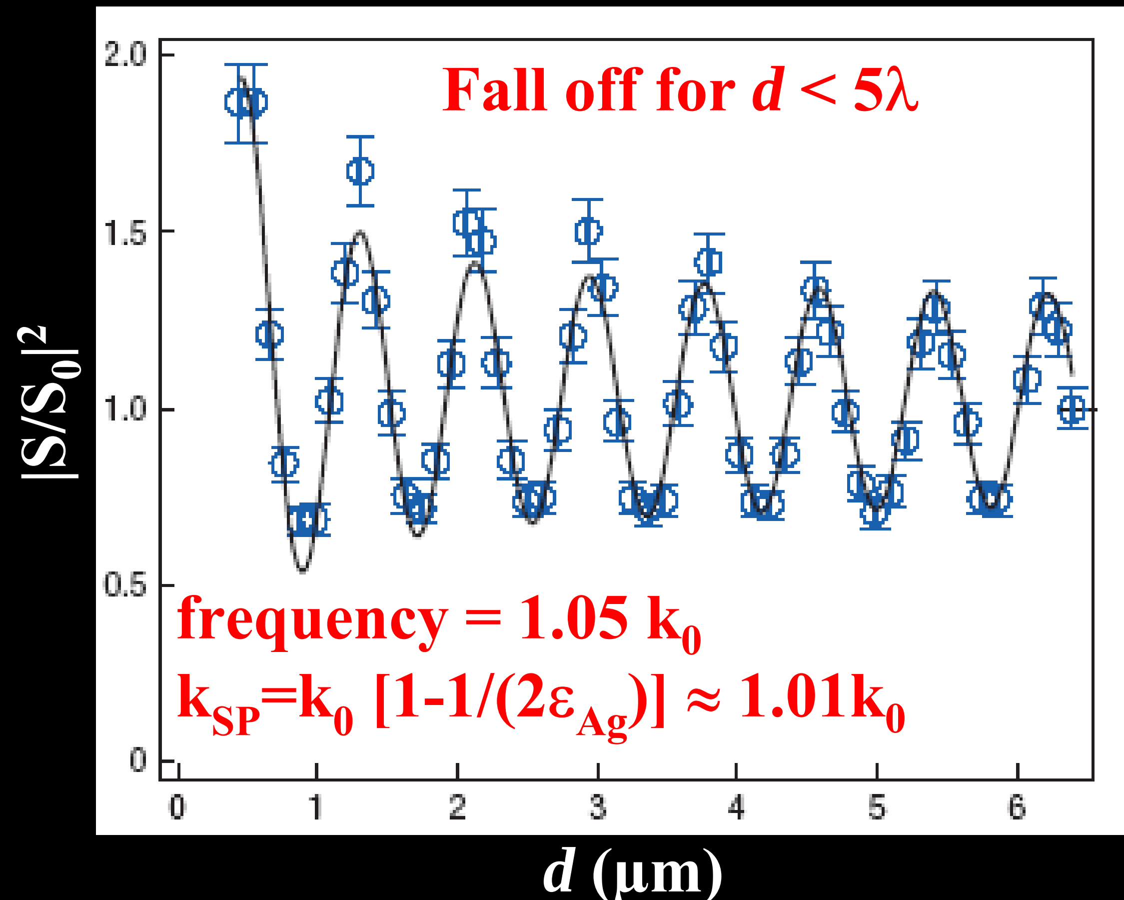
Slit-Groove experiment

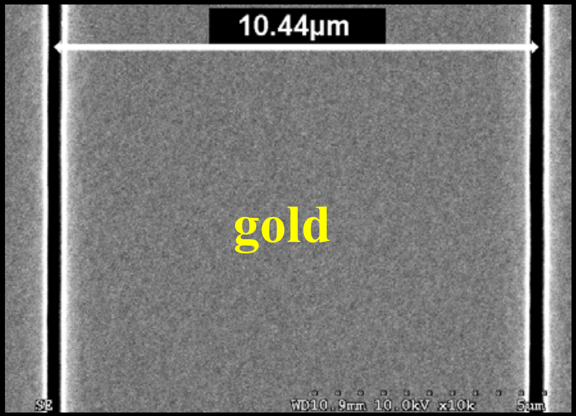






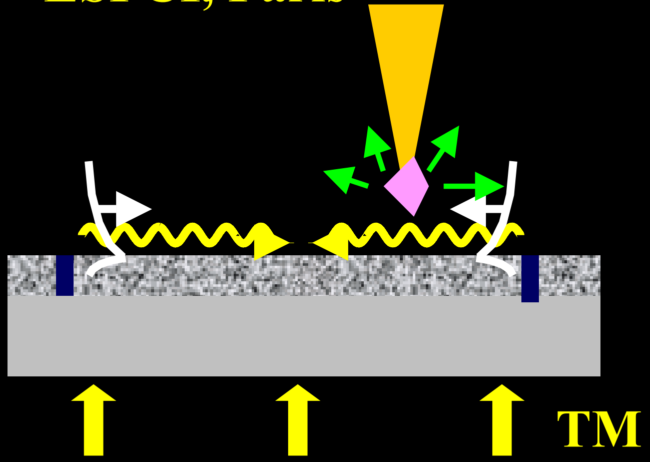
G. Gay et al. Nature Phys. 2, 262 (2006) → promote an other model than SPP & quasi-CW (CDEW model)

Near field validation

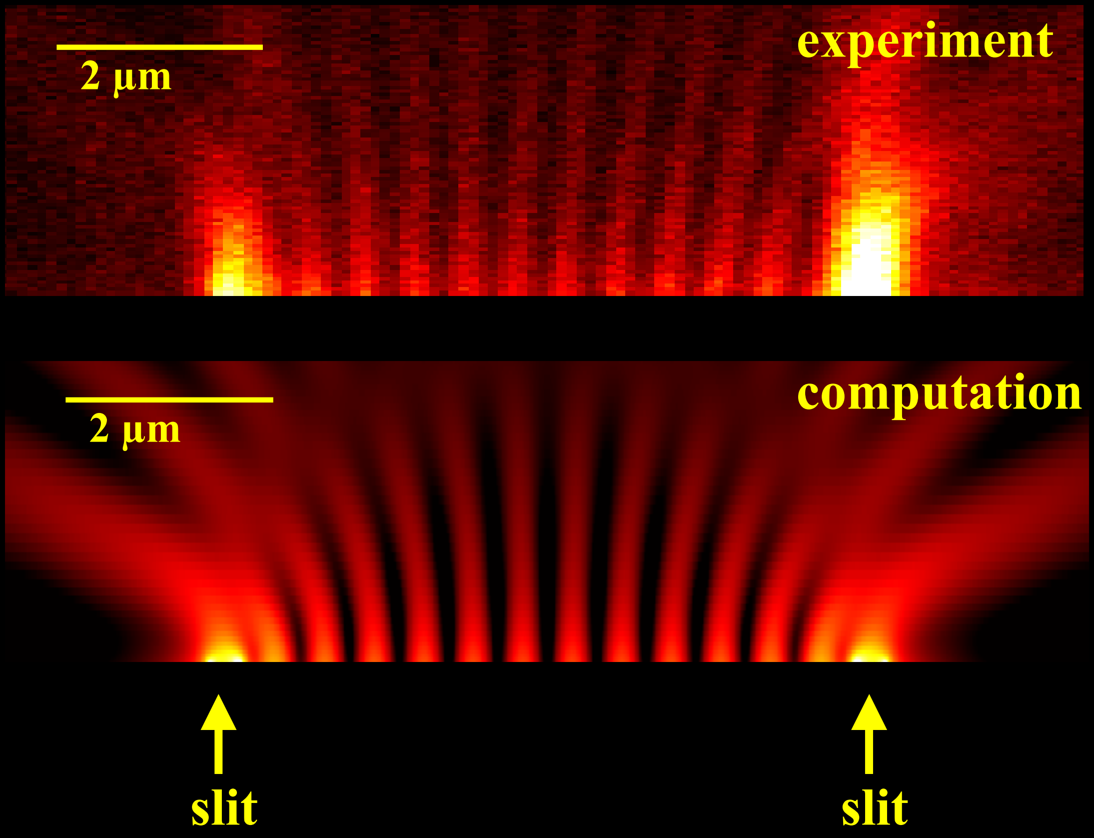


G. JULIE, V. MATHET
IEF, Orsay

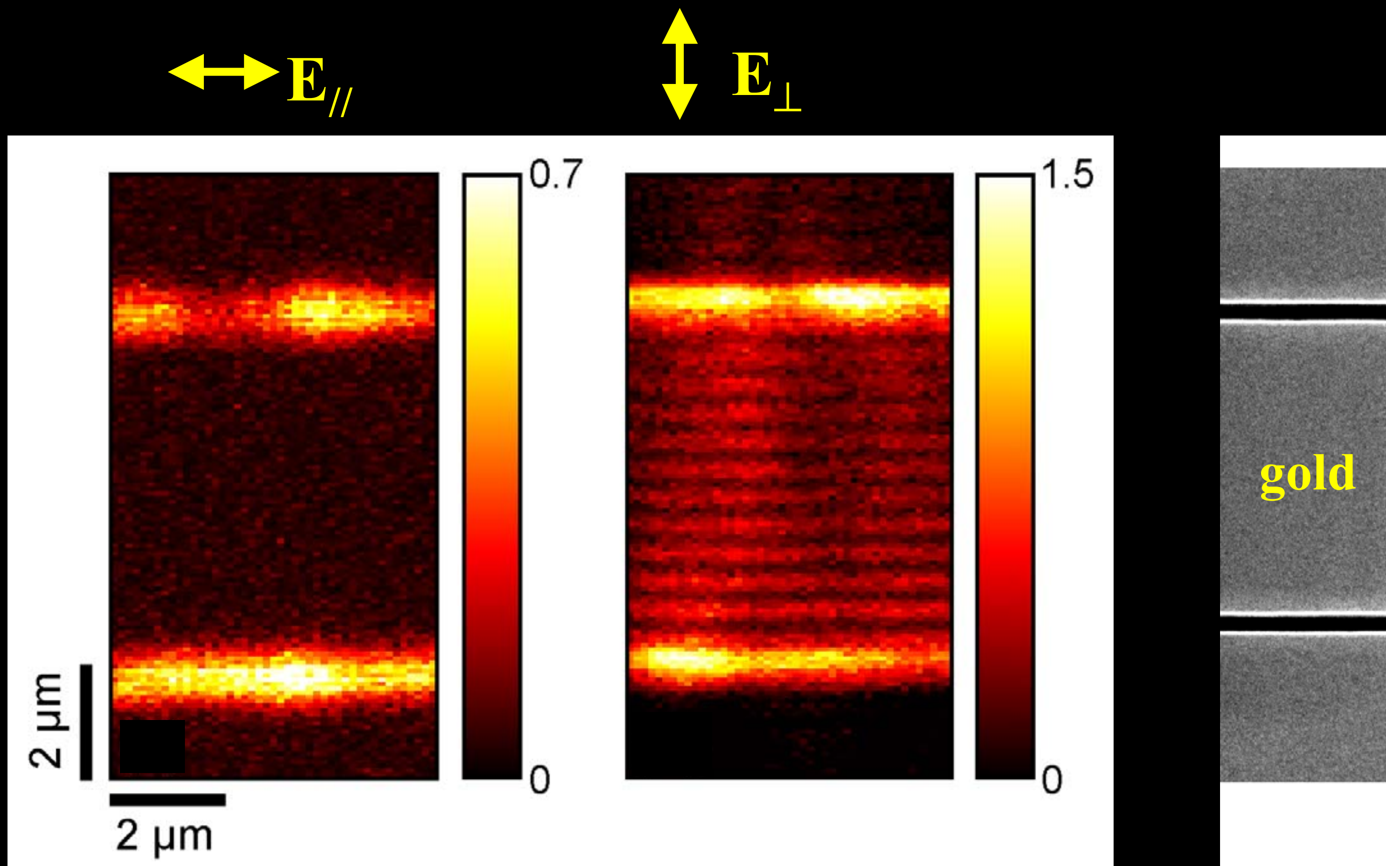
L. AIGOUY
ESPCI, Paris



λ=975 nm



Field recorded on the surface



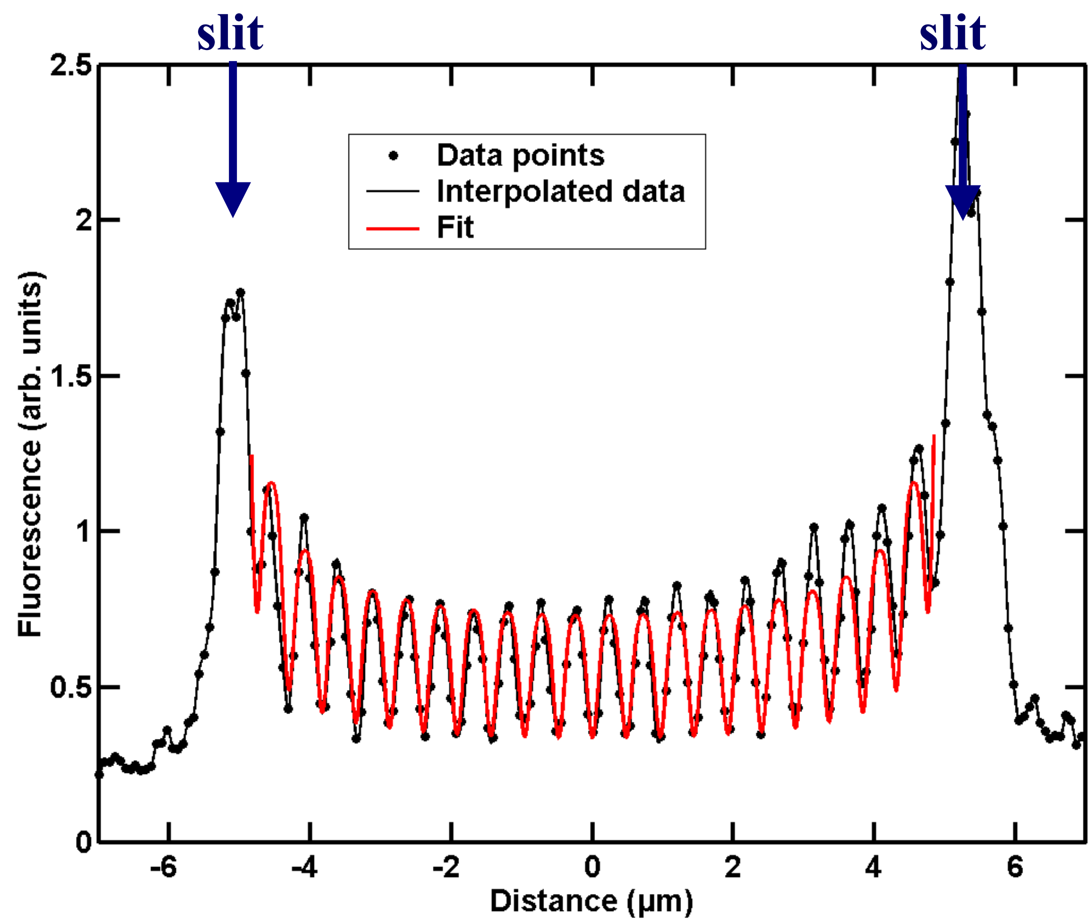
λ=974 nm

$$E_z = A_{SP}\sin(k_{SP}x) + A_c\frac{\exp(ik_0x)}{(x+d)^m}[ik_0 - m/(x+d)] - A_c\frac{\exp(-ik_0x)}{(x-d)^m}[ik_0 + m/(x-d)]$$

standing SPP

right-traveling cylindrical wave

left-traveling cylindrical wave

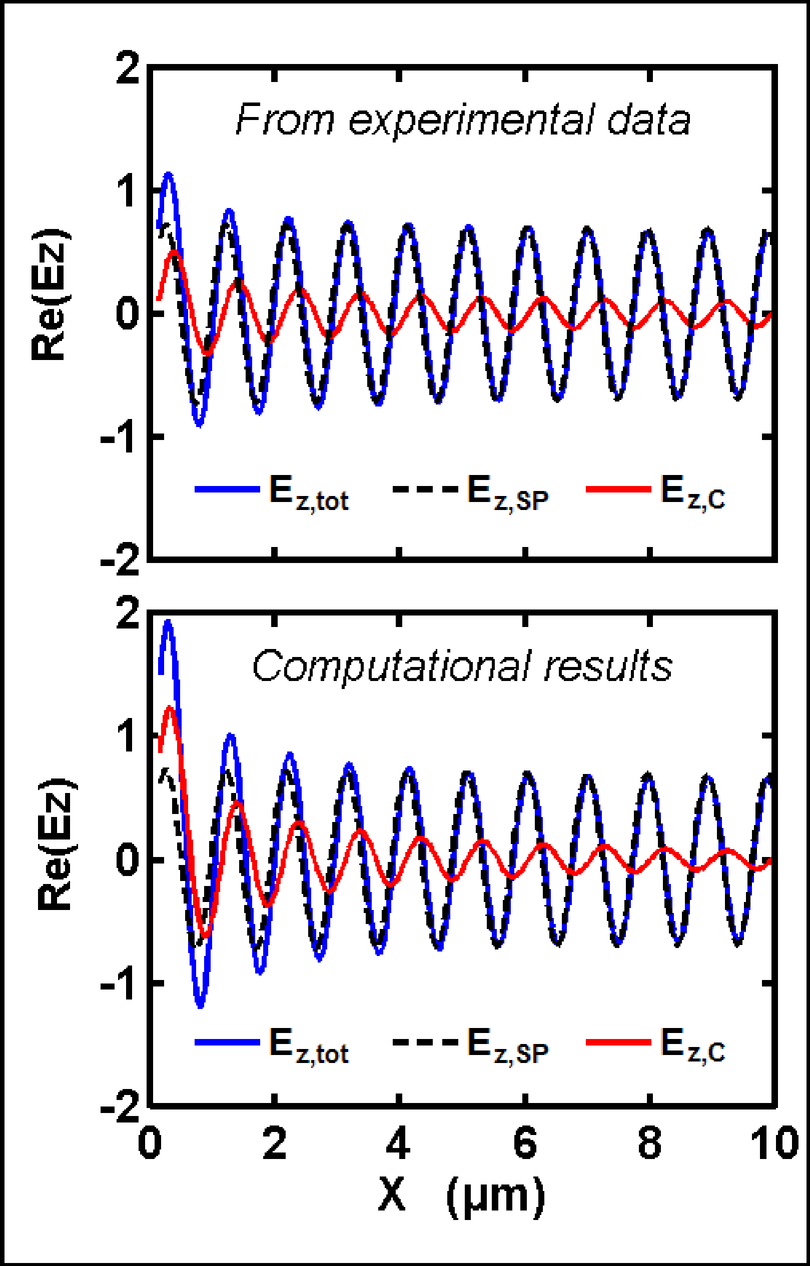


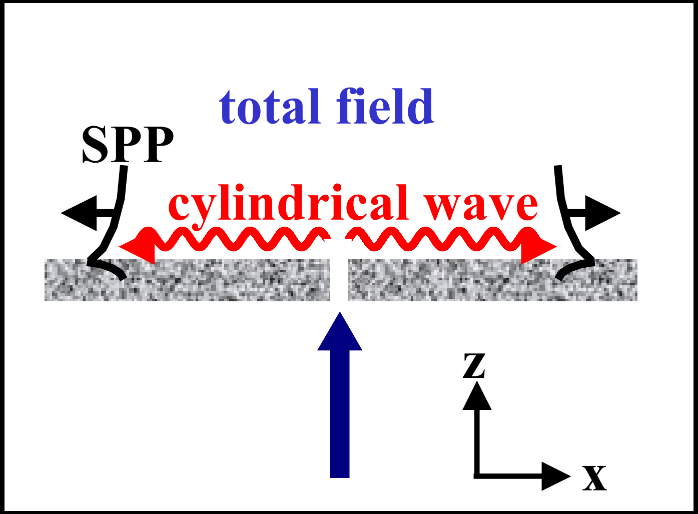
fitted parameter

A_{SP} (real)

A_c (complex)

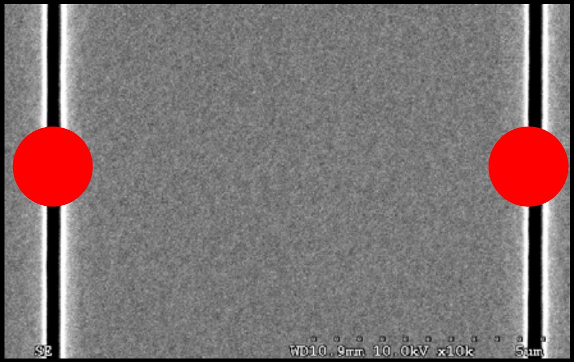
(m=0.5)





L. Aigouy et al., PRL 98, 153902 (2007)

A direct observation?



If one controls the two beam intensity and phase (or the separation distance) so that there is no SPP generated on the right side, do I observe a pure quasi-CW on the right side of the doublet?

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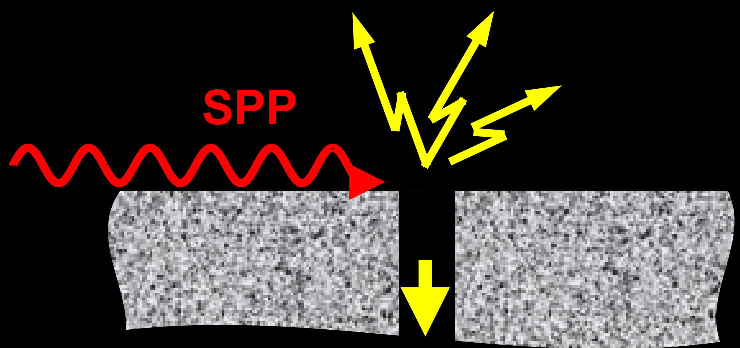
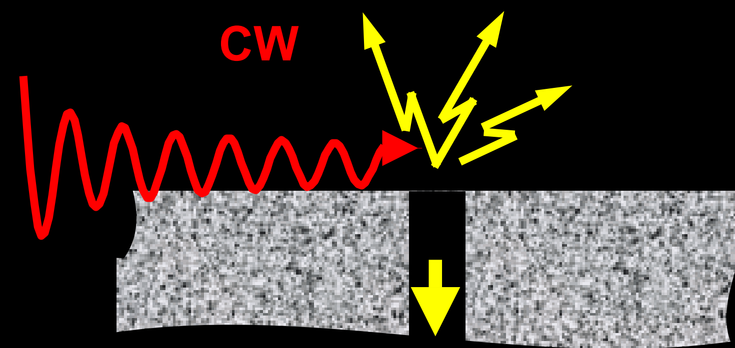
Defining scattering coefficients for CWs

The SPP is a normal mode





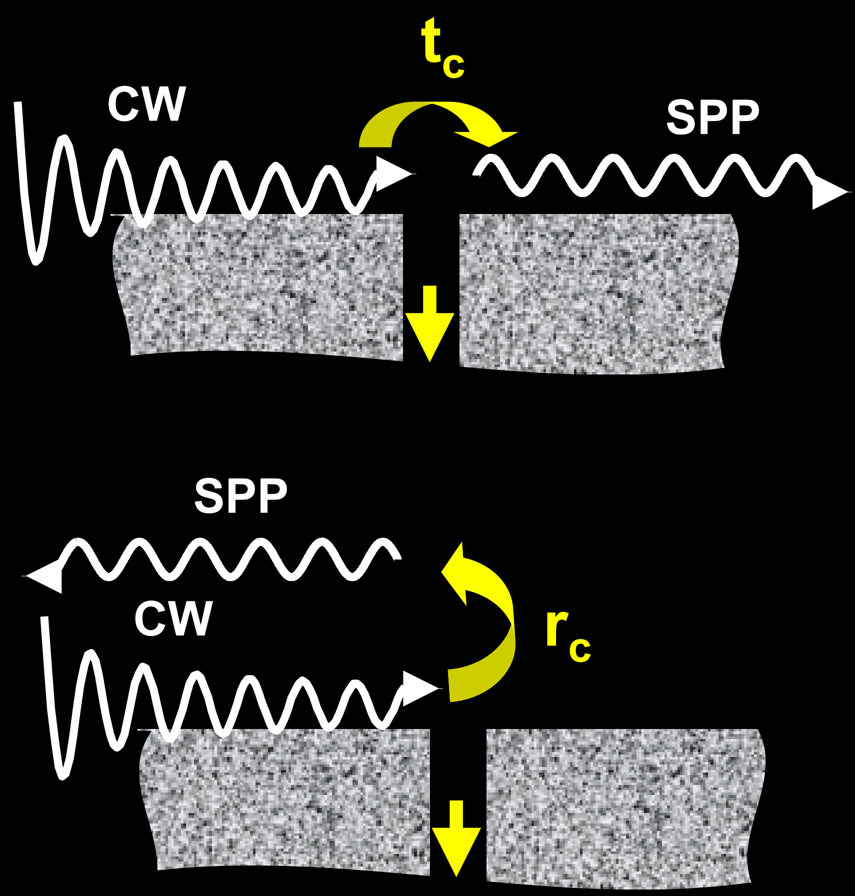
⇒ elastic scattering coefficients like the SPP transmission may be easily defined.

You may also define inelastic scattering coefficients with other modes like the radiated plane waves

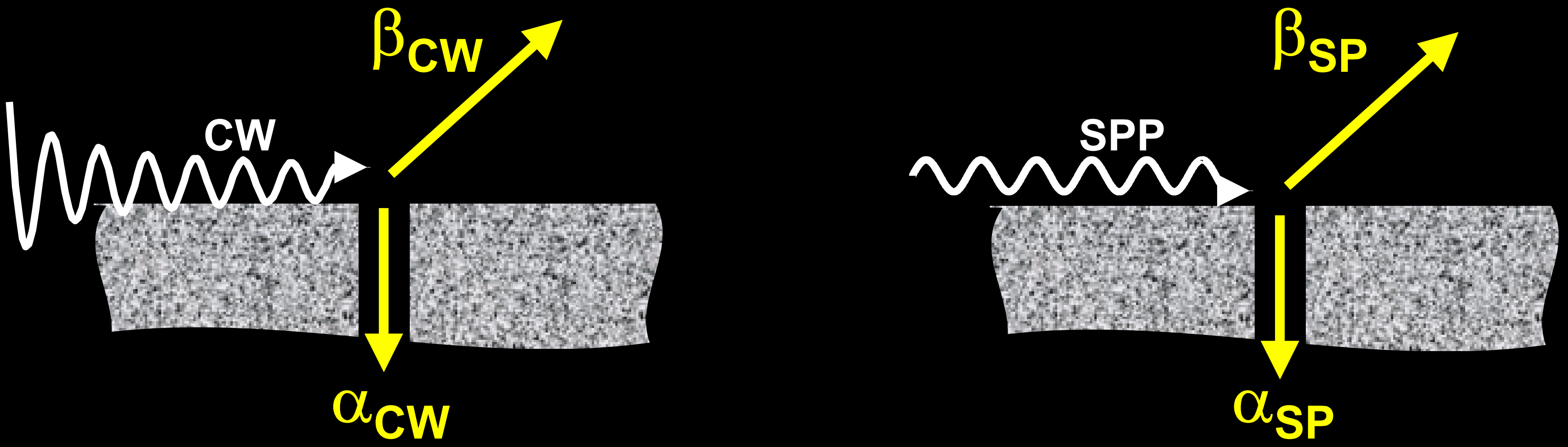
You may use mode orthogonality and reciprocity relationships.

?

CW-to-SPP cross-conversion



Other scattering coefficients



$$\beta_{CW} = \beta_{SP}$$
$$\alpha_{CW} = \alpha_{SP}$$

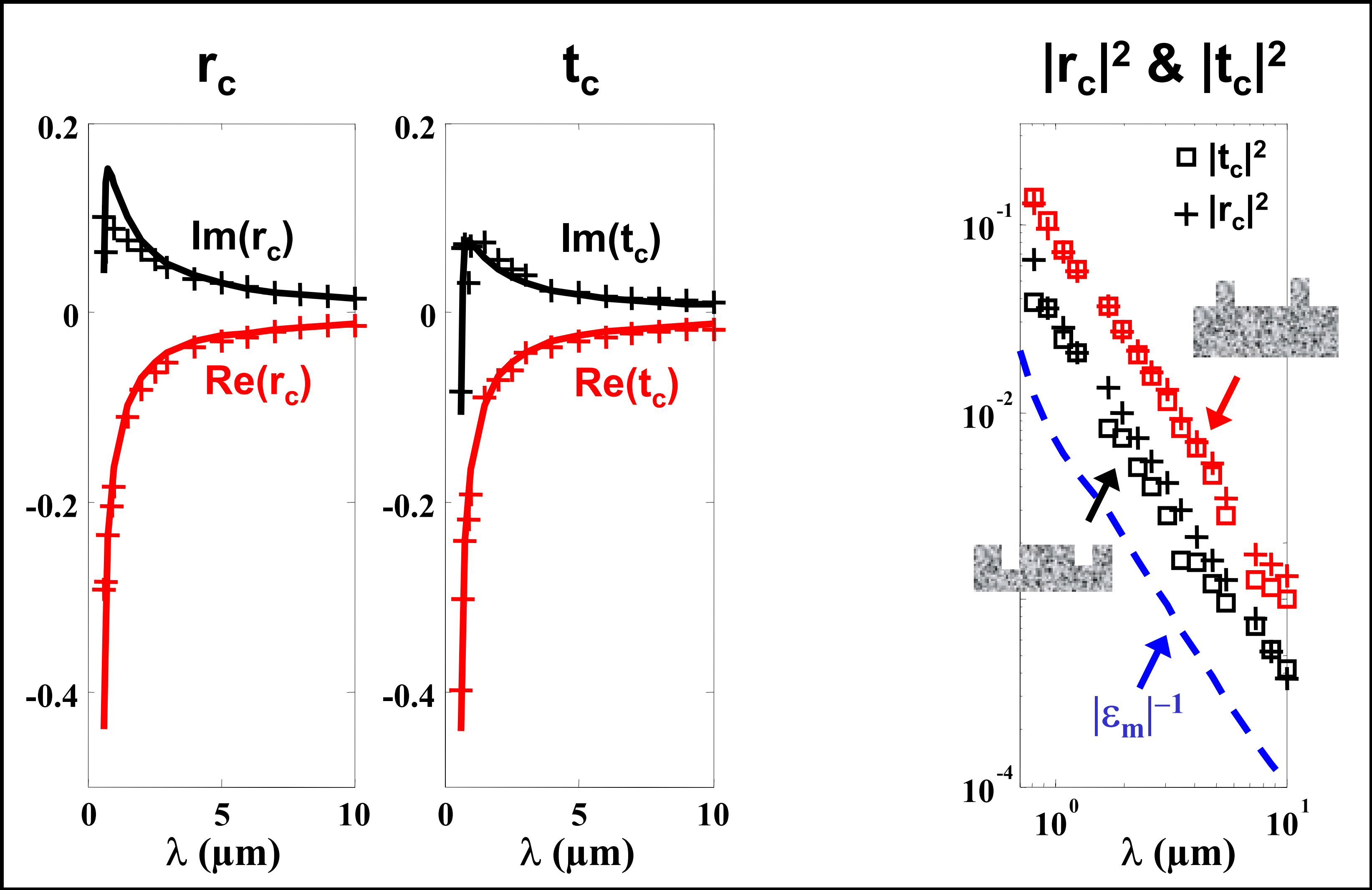
X. Yang et al., Phys. Rev. Lett. 102, 153903 (2009)

Cross-conversion scattering coefficients

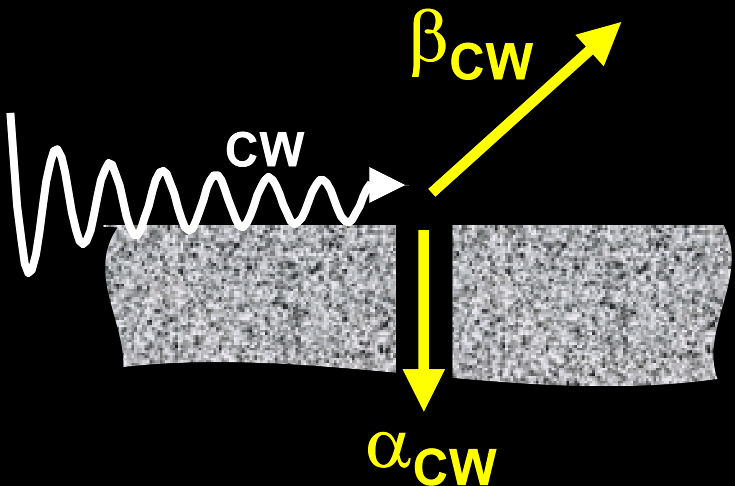
Ansatz : THE SCATTERED FIELDS ARE IDENTICAL.
(if the two waves are normalized so that they have identical amplitudes at the slit)

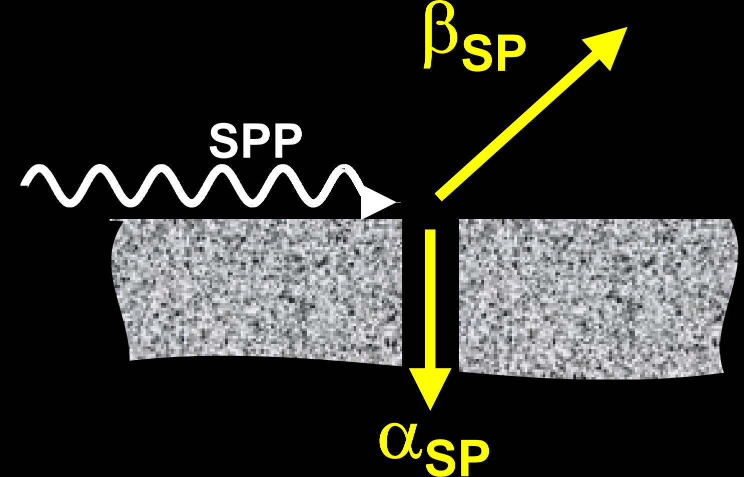
X. Yang et al., Phys. Rev. Lett. 102, 153903 (2009)

Scaling law for r_c and t_c



Other scattering coefficients





$$\beta_{CW} = \beta_{SP}$$
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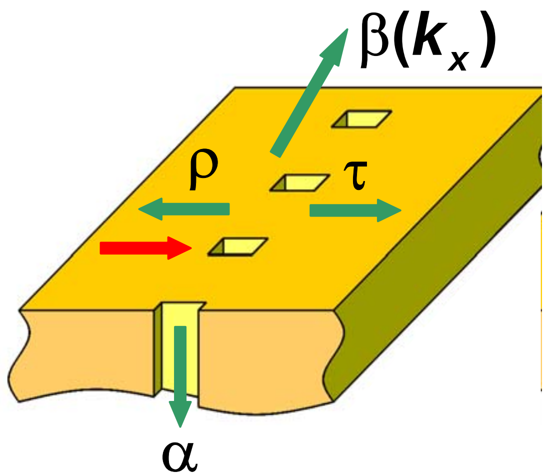
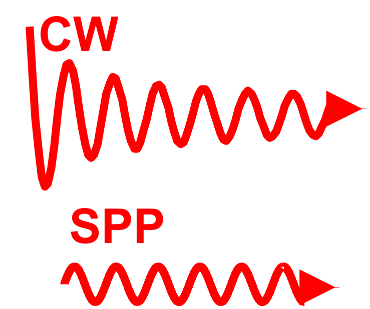
X. Yang et al., Phys. Rev. Lett. 102, 153903 (2009)

Mixed SPP-CW model for the extraordinary optical transmission



pure SPP model

$$t_A = t + \frac{2\alpha\beta}{u^{-1} - (\rho+\tau)} \qquad r_A = \frac{2\alpha^2}{u^{-1} - (\rho+\tau)}$$



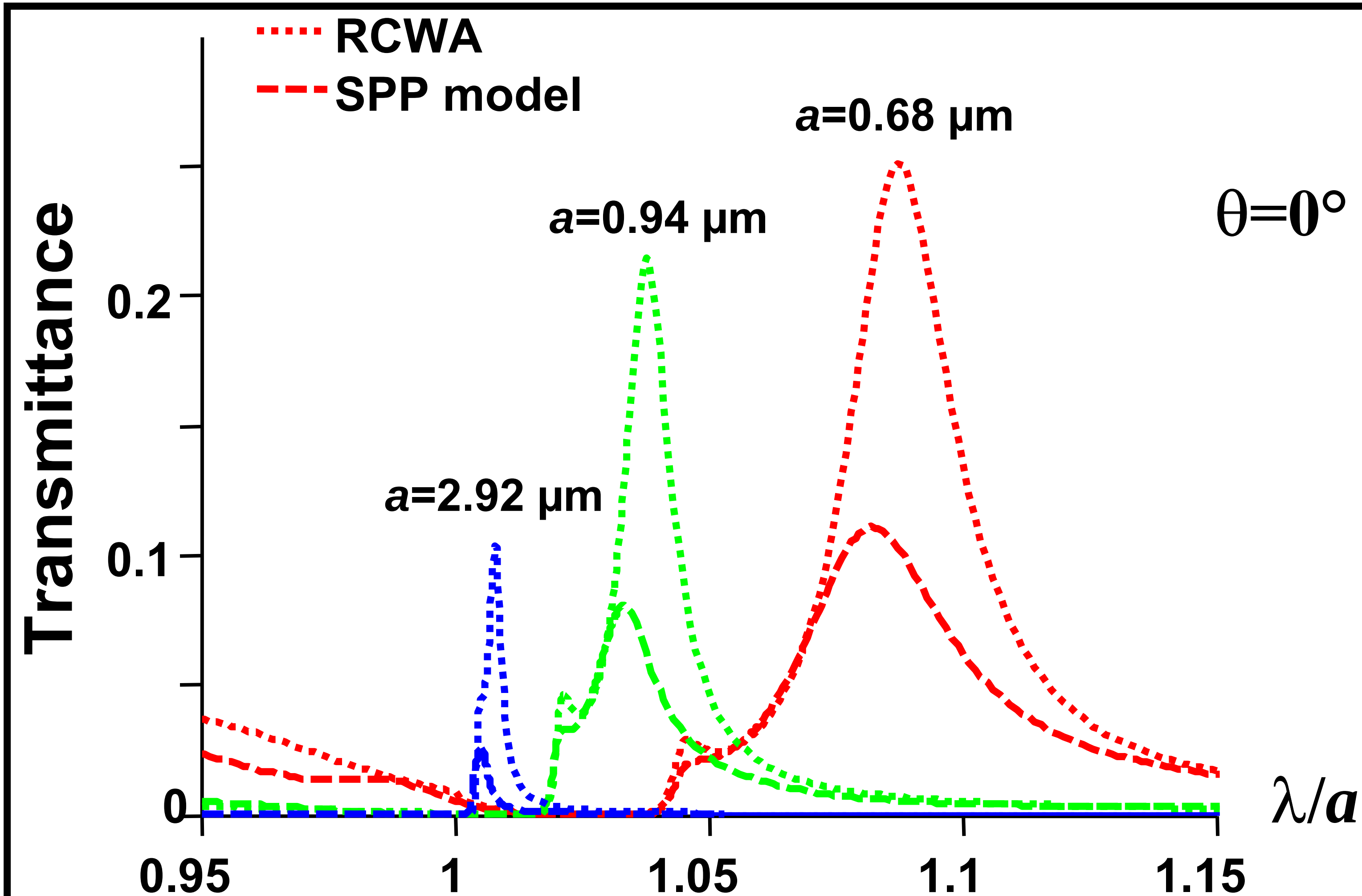
mixed SPP-CW model

$$t_A = t + \frac{2\alpha\beta}{(P^{-1}+1) - (\rho+\tau)} \qquad r_A = \frac{2\alpha^2}{(P^{-1}+1) - (\rho+\tau)}$$

$$P = 1/(u^{-1} - 1) + \Sigma_{n=1,\infty} H_{CW}(na)$$

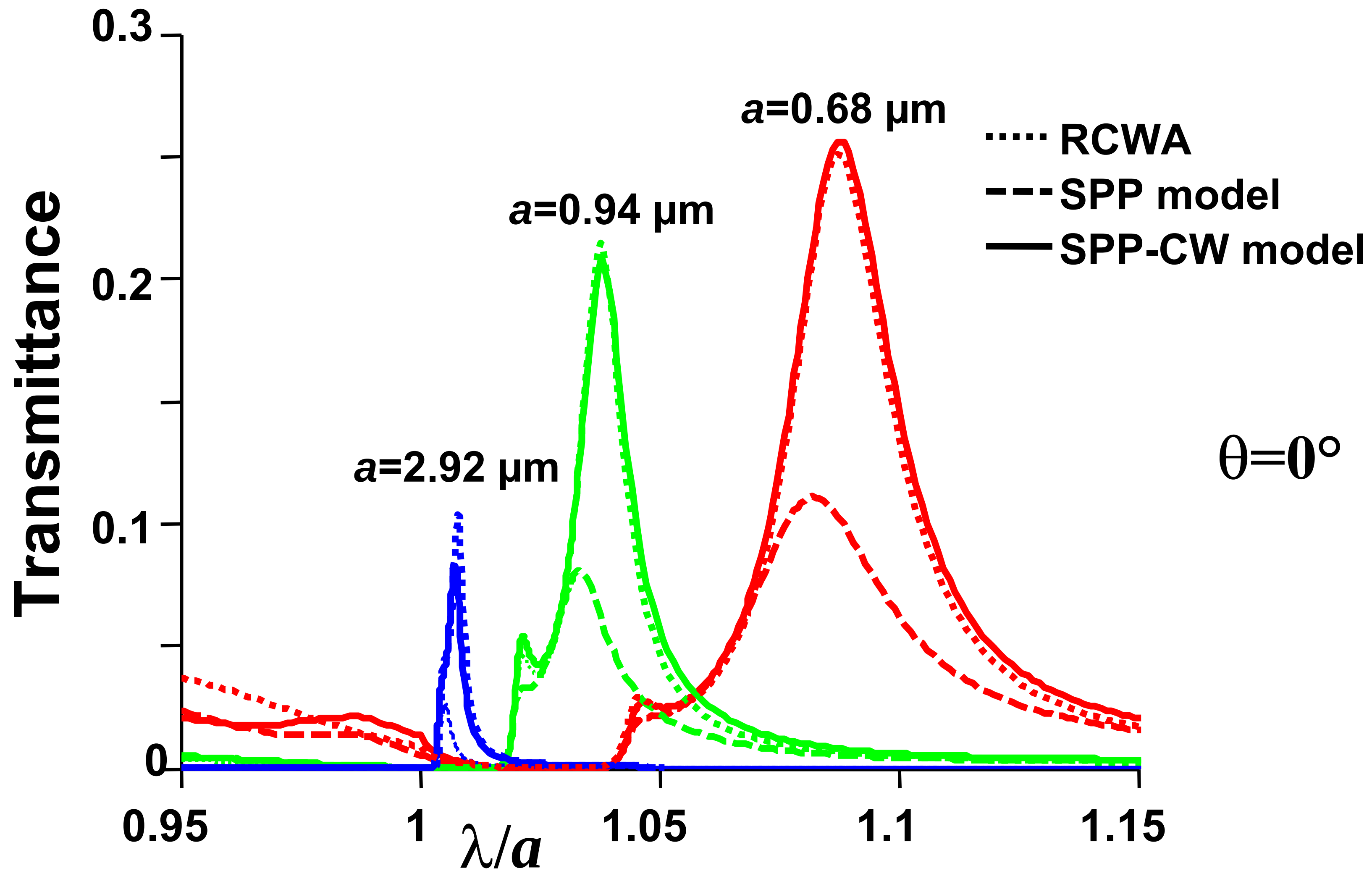
H. Liu et al. (submitted)

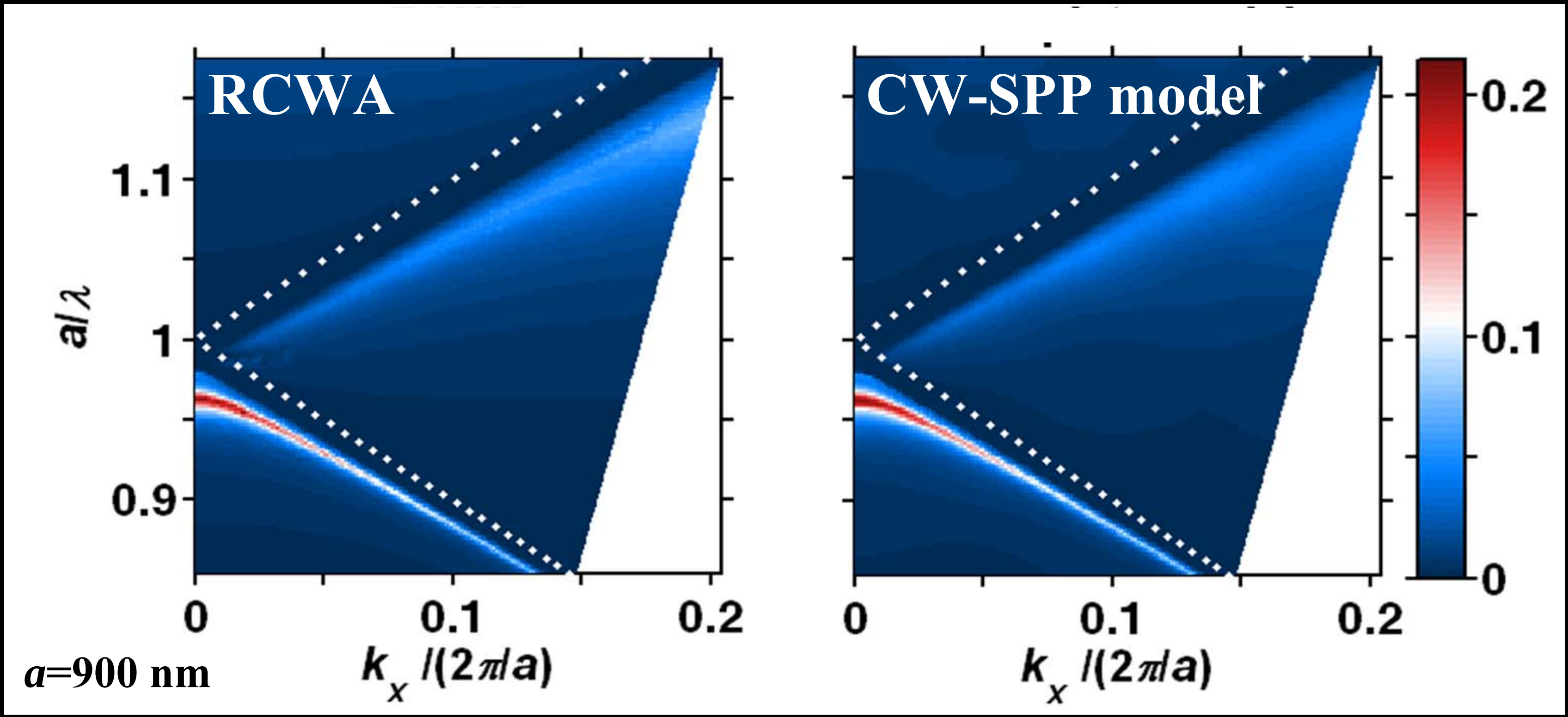
Pure SPP-model prediction of the EOT

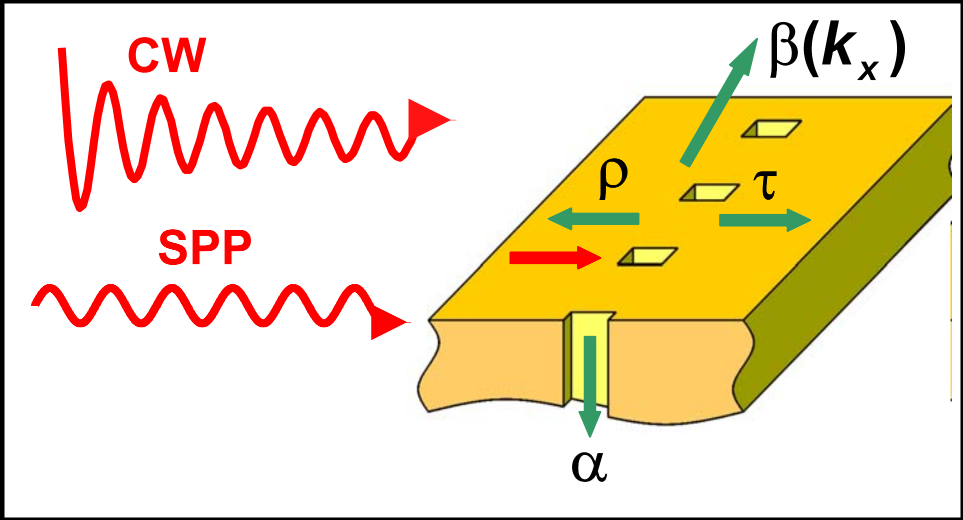


H. Liu & P. Lalanne, Nature 452, 448 (2008).

Mixed SPP-CW model for the extraordinary optical transmission

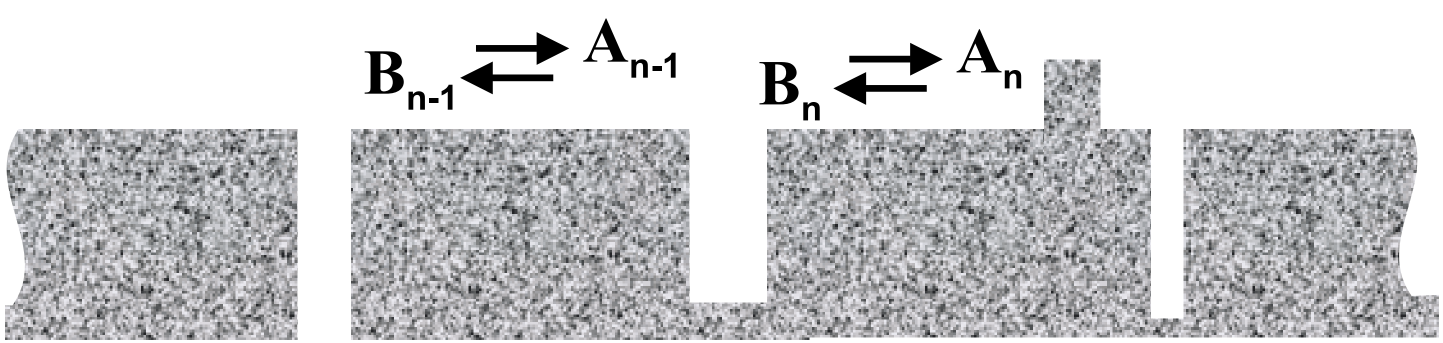



RCWA
CW-SPP model
a/λ
1.1
1
0.9
0
0.1
0.2
k_x /(2π/a)
a=900 nm
0.2
0.1
0


CW
SPP
β(k_x)
ρ
τ
α

SPP+quasi-CW coupled-mode equations

It is not necessary to be periodic
It is not necessary to deal with the same indentations



Conclusion

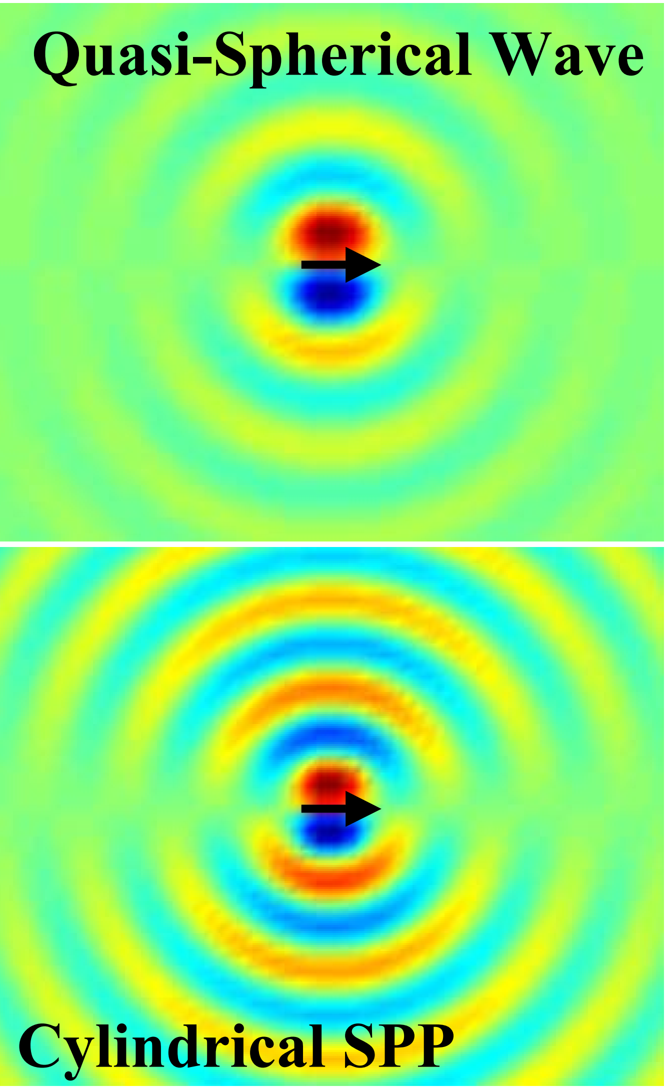
Two different microscopic waves, the SPP mode and the quasi-CW, are at the essence of the rich physics of metallic sub-λ surfaces

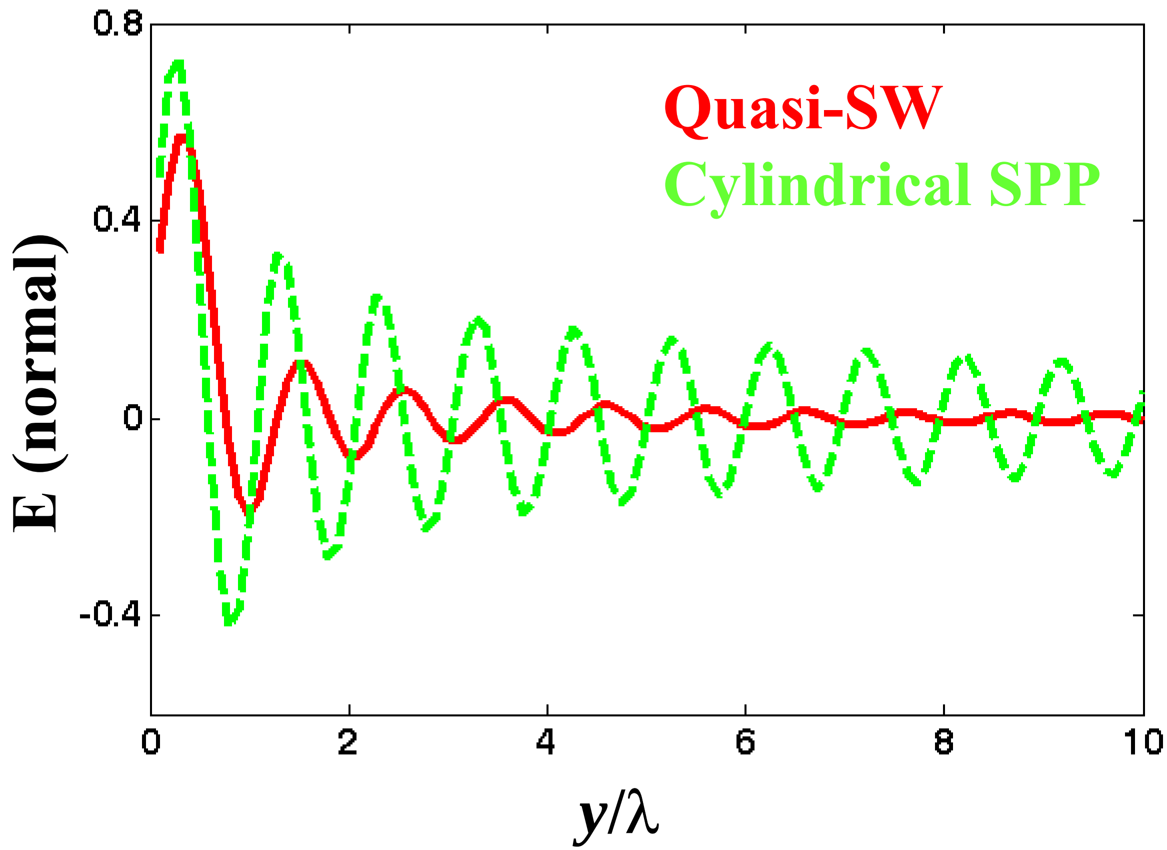
Their relative weights strongly vary with the metal permittivity

They echange their energy through a cross-conversion process, whose efficiency scales as $|\varepsilon_m|^{-1}$

The local SPP elastic or inelastic scattering coefficients are essential to understand the optical properties, since they apply to both the SPP and the quasi-CW

SPP, quasi-CW, C-SPP, quasi-SP





gold @ λ=800 nm