Sub-λ metallic surfaces: a microscopic analysis

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Surface plasmon polariton

SPPs are localized electromagnetic modes/ charge density oscillations at the interfaces, which exponentially decay on both sides.

\[ k_{SP} = \frac{\omega}{c} \left( \frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m} \right)^{1/2} \]
Surface plasmon polariton

\[ \exp(-z/\delta_1) \]
\[ \delta_1 = \lambda \varepsilon^{1/2}/2\pi \gg \lambda \]

\[ \exp(-z/\delta_2) \]
\[ \delta_2 = \lambda \varepsilon^{-1/2}/2\pi \approx cte \]

Drude model: \[ |\varepsilon| \propto \lambda^2 \]
1. The emblematic example of the EOT
- extraordinary optical transmission (EOT)
- limitation of classical "macroscopic" grating theories
- a microscopic pure-SPP model of the EOT

2. SPP generation by 1D sub-\( \lambda \) indentation
- rigorous calculation (orthogonality relationship)
- slit example
- scaling law with the wavelength

3. The quasi-cylindrical wave
- definition & properties
- scaling law with the wavelength

4. Multiple Scattering of SPPs & quasi-CWs
- definition of scattering coefficients for the quasi-CW
The extraordinary optical transmission

transmittance ($\omega, k_{//}$)

1/$\lambda$ ($\mu$m$^{-1}$)

minimum

EOT peak

SPP of the flat interface

Two branches
The extraordinary optical transmission

Black: experiment  red: Fano fit
The extraordinary optical transmission

The extraordinary optical transmission = a grating scattering problem
What is learnt from grating theory?

Hertz (1888) first used a wire grid polarizer for testing the newly discovered radio wave.

Nearly 100% of the incident energy is transmitted at resonance frequencies for TM polarization.
Poles and zeros of the scattering matrix

Global analysis.

-Why does the pole exist?
-Why does the zero exist?
-Why are they close or not to the real axis?

E. Popov et al., PRB 62, 16100 (2000).
The surface-mode interpretation

Resonance-assisted tunneling

\[ t_F = \frac{t_A^2 \exp(ik_0nd)}{1 - r_A^2 \exp(2ik_0nd)} \]

The surface-mode interpretation

\[ \frac{1}{\lambda} \]

Mode of the perforated interface = pole of \( r_A \) or \( t_A \)

The surface-mode interpretation

Resonance-assisted tunneling

\[ t_F = \frac{t_A^2 \exp(ik_0nd)}{1 - r_A^2 \exp(2ik_0nd)} \]


\[ \rightarrow \text{reinforcement of the initial vision of a SPP assisted effect} \]
The surface-mode also exists as $\lambda \to \infty$.

Theory:

Experimental verification:
Weaknesses of classical grating theories

Resonance-assisted tunneling

\[ t_F = \frac{t_A^2 \exp(i k_0 n d)}{1 - r_A^2 \exp(2i k_0 n d)} \]


- The resonance of \( t_F \) is explained by the resonance of another scattering coefficients.
- In reality, nothing is known about the waves that are launched in between the hole and that are responsible for the EOT.
\[ r \propto \frac{\lambda - \lambda'^2}{\lambda - \lambda' p} \]

\[ R(\theta_0, \lambda_0) = 0 \]

Microscopic pure-SPP model

SPP coupled-mode equations

\[ A_n = w_1 w_2 \ldots w_n \beta(k_x) + u_n \tau A_{n-1} + u_n \rho B_{n+1} \]
\[ B_n = w_1 w_2 \ldots w_n \beta(-k_x) + u_{n+1} \tau B_{n-1} + u_n \rho A_{n-1} \]
\[ c_n = w_1 w_2 \ldots w_n t(-k_x) + u_n \alpha A_{n-1} + u_{n+1} \alpha B_{n+1} \]

with \( u_n = \exp(ik_{SP}a_n) \), \( w_n = \exp(ik_xa_n) \)
Microscopic pure-SPP model

\[ t_F = \frac{t_A^2 \exp(ik_0nd)}{1 - r_A^2 \exp(2ik_0nd)} \]

\[ t_A = t + \frac{2\alpha \beta}{u^{-1} - (\rho + \tau)} \quad r_A = \frac{2\alpha^2}{u^{-1} - (\rho + \tau)} \]

only non-resonant quantities
Microscopic interpretation

SPP coupled-mode equations ($k_x = 0$)

$$t_A = t + \frac{2\alpha\beta}{u^{-1} - (\rho + \tau)}$$

$$r_A = r + \frac{2\alpha^2}{u^{-1} - (\rho + \tau)}$$

$|u| \approx 1$

$u = \exp(ik_{sp}a) \rightarrow |u|^{-1}$ slightly larger than 1

$|\tau| \approx 1$

$\tau$ slightly smaller than 1

Resonance condition

$\text{Re}(k_{sp})a + \text{arg}(\tau) \approx 0 \mod 2\pi$
holes \[ \text{slits} \]

\[ t_F = \frac{t_A^2 \exp(ik_0nd)}{1 - r_A^2 \exp(2ik_0nd)} \]

- \( n \) complex
- \(|\exp(2ik_0nd)| \ll 1\)
- pole of \( t_F \) = pole of \( r_A \) (for \( k_0d >> 1 \))

- \( n \) real
- \(|\exp(2ik_0nd)| = 1\)
- no relation between the pole of \( t_F \) and that of \( r_A \)
- FP condition:
  \[ \arg(r_A) + k_0nd = 2m\pi \]
\[ t_F = \frac{t_A^2 \exp(ik_0nd)}{1 - r_A^2 \exp(2ik_0nd)} \]

\[ |r_A| \]

\[
\begin{array}{c}
\begin{array}{c}
\lambda /a \\
0 \quad 10 \quad 20 \quad 30
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\begin{array}{c}
\lambda /a \\
0 \quad 1 \quad 1.2
\end{array}
\end{array}
\]
Microscopic interpretation

SPP coupled-mode equations ($k_x=0$)

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Microscopic interpretation

resonance condition:

$$\text{Re}(k_{SP})a + \text{arg}(\tau) \approx k_x a \quad (\text{modulo } 2\pi)$$

the macroscopic surface Bloch mode

superposition of many elementary SPPs scattered by individual hole chains that fly over adjacent chains and sum up constructively
Influence of the metal conductivity

\[ \theta = 0^\circ \]

Transmittance

\( \frac{\lambda}{a} \)

- Red dotted line: RCWA
- Red dashed line: SPP model

\( a = 0.68 \mu m \)

\( a = 0.94 \mu m \)

\( a = 2.92 \mu m \)

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   - definition of scattering coefficients for the quasi-CW
How to know how much SPP is generated?
1) make use of the completeness theorem for the normal modes of waveguides

\[ H_y = \left( \alpha^+(x) + \alpha^-(x) \right) H_{SP}(z) + \sum_{\sigma} a_{\sigma}(x) H^{(\text{rad})}_{\sigma}(z) \]

\[ E_z = \left( \alpha^+(x) - \alpha^-(x) \right) E_{SP}(z) + \sum_{\sigma} a_{\sigma}(x) E^{(\text{rad})}_{\sigma}(z) \]

2) Use orthogonality of normal modes

\[ \int_{-\infty}^{\infty} dz \ H_y(x,z) \ E_{SP}(z) = 2 \left( \alpha^+(x) + \alpha^-(x) \right) \]

\[ \int_{-\infty}^{\infty} dz \ E_z(x,z) \ H_{SP}(z) = 2 \left( \alpha^+(x) - \alpha^-(x) \right) \]

PL, J.P. Hugonin and J.C. Rodier, PRL 95, 263902 (2005)
General theoretical formalism

\[ \exp[-\text{Im}(k_{sp}x)] \]

[Graph showing $|\alpha^-|^2$ and $|\alpha^+|^2$ as functions of $x/\lambda$.]
General theoretical formalism applied to gratings
General theoretical formalism applied to grooves ensembles

- 11-groove optimized SPP coupler with 70% efficiency.
- The incident gaussian beam has been removed.
General theoretical formalism

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General theoretical formalism

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SPP generation

$n_1$

$n_2$
SPP generation

\[ \alpha^-(x) \rightarrow n_1 \rightarrow \alpha^+(x) \]

\[ n_2 \]
SPP generation

$\beta^-(x)$

$n_1$

$n_2$

$\beta^+(x)$
1) assumption: the near-field distribution in the immediate vicinity of the slit is weakly dependent on the dielectric properties

2) Calculate this field for the PC case

3) Use orthogonality of normal modes

\[
\int_{-\infty}^{\infty} dz \, H_y(x,z) E_{sp}(z) = 2 (\alpha^+(x) + \alpha^-(x)) \\
\int_{-\infty}^{\infty} dz \, E_x(x,z) H_{sp}(z) = 2 (\alpha^+(x) - \alpha^-(x))
\]

Valid only at \( x = \pm w/2 \) only!

\[
\alpha^+ = \alpha^- = \left( \frac{4 \, n_2}{\pi \, n_1} \frac{\sqrt{\text{\varepsilon}}}{\varepsilon + n_2^2} \right)^{1/2} \sqrt{\frac{w}{\lambda}} I_1 \frac{\sqrt{\frac{w}{\lambda} I_1}}{1 + (n_2/n_1) w' I_0}
\]

Question: how is it possible that with a perfect metallic case, one gets accurate results?
describe geometrical properties
-the SPP excitation peaks at a value $w = 0.23\lambda$ and for visible frequency, $|\alpha|^2$ can reach 0.5, which means that of the power coupled out of the slit half goes into heat

1) assumption: the near-field distribution in the immediate vicinity of the slit is weakly dependent on the dielectric properties.

2) Calculate this field for the PC case.

3) Use orthogonality of normal modes.

\[
\alpha^+ = \alpha^- = \left( \frac{4}{\pi} \frac{n_2}{n_1} \frac{\sqrt{\varepsilon}}{\varepsilon + n_2^2} \right)^{1/2} \sqrt{\frac{w}{\lambda}} \frac{I_1}{1 + (n_2/n_1) w' I_0}
\]

describe material properties

- Immersing the sample in a dielectric enhances the SP excitation \((\propto n_2/n_1)\).

- The SP excitation probability \(|\alpha^+|^2\) scales as \(|\varepsilon(\lambda)|^{-1/2}\).

$S = \beta + [t_0 \alpha \exp(2ik_0 n_{\text{eff}}h)] / [1-r_0 \exp(2ik_0 n_{\text{eff}}h)]$

groove

\[
S = \beta + \frac{[t_0 \alpha \exp(2ik_0n_{\text{eff}}h)]}{[1-r_0 \exp(2ik_0n_{\text{eff}}h)]}
\]

\[
\begin{align*}
\lambda &= 0.8 \, \mu \text{m} \\
\lambda &= 1.5 \, \mu \text{m}
\end{align*}
\]

Can be much larger than the geometric aperture!
Grooves

\[ S = \beta + \left[ t_0 \alpha \exp(2ik_0n_{\text{eff}}h) \right] / \left[ 1-r_0 \exp(2ik_0n_{\text{eff}}h) \right] \]

\[ r = r_{\infty} + \alpha^2 \exp(2ik_0n_{\text{eff}}h) / \left[ 1-r_0 \exp(2ik_0n_{\text{eff}}h) \right] \]

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- All dimensions are scaled by a factor $G$
- The incident field is unchanged $H_{\text{inc}}(\lambda) = H_{\text{inc}}(\lambda')$

If $\varepsilon = \varepsilon'$, the scattered field is unchanged: $E'(r') = E(r'/G)$

The difficulty comes from the dispersion: $\varepsilon' \neq \varepsilon$

$\varepsilon'/\varepsilon = G^2$ (for Drude metals)
A perturbative analysis fails

Unperturbed system
\[
\begin{align*}
\nabla \times E' &= j \omega \mu_0 H' \\
\nabla \times H' &= -j \omega \varepsilon(r) E'
\end{align*}
\]

Perturbed system
\[
\begin{align*}
\nabla \times E &= j \omega \mu_0 H \\
\nabla \times H &= -j \omega \varepsilon(r) E - j \omega \Delta \varepsilon E
\end{align*}
\]

\[E_s = E - E' \quad \text{and} \quad H_s = H - H'\]
\[
\begin{align*}
\nabla \times E_s &= j \omega \mu_0 H_s \\
\nabla \times H_s &= -j \omega \varepsilon(r) E_s - j \omega \Delta \varepsilon E
\end{align*}
\]

Indentation acts like a volume current source: \[J = -j \omega \Delta \varepsilon E\]
A perturbative analysis fails

One may find how $c_1$ or $c_{-1}$ scale.
"Easy" task with the reciprocity

Source-mode reciprocity

\[ c_1 = -E_{SP}^{(-1)}(r_0) \cdot J \]
\[ c_{-1} = -E_{SP}^{(1)}(r_0) \cdot J \]

provided that the SPP pseudo-Poynting flux is normalized to 1

\[ \frac{1}{2} \int dz \left[ E_{SP}^{(1)} \times H_{SP}^{(1)} \right] \cdot x = 1 \]

\[ E_{SP} = N^{1/2} \exp(ik_{SP}x)\exp(i\gamma_{SP}z), \text{ with } N \approx |\varepsilon|^{1/2}/(4\omega\varepsilon_0) \]
A perturbative analysis fails

One may find how $c_1$ or $c_{-1}$ scale. Then, one may apply the first-order Born approximation ($E = E' \approx E_{\text{inc}} / 2$). Since $E_{\text{inc}} \approx 1$, one obtains that dielectric and metallic indentations scale differently.

$\omega \Delta \varepsilon E \delta(r-r_0)$

$\Delta \varepsilon \sim 1$ for dielectric ridges
$\Delta \varepsilon \sim \varepsilon$ for dielectric grooves
$\Delta \varepsilon \sim \varepsilon$ for metallic ridges
Numerical verification

$$|H_{Sp}|$$

$$\varepsilon^{-1/2}$$

$$\lambda (\mu m)$$

H. Liu et al., IEEE JSTQE 14, 1522 (2009 special issue) (results obtained for gold)
Numerical verification

\[ |H_{Sp}| \]

\[ \lambda (\mu m) \]

\[ \varepsilon^{-1/2} \]

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Numerical verification

$|H_{Sp}|$

$|\varepsilon|^{-1/2}$

$\lambda (\mu m)$

H. Liu et al., IEEE JSTQE 14, 1522 (2009 special issue) (results obtained for gold)
Local field corrections: small sphere ($R \ll \lambda$)

\[ \Delta \varepsilon = \varepsilon_h - \varepsilon_b \]

J.D. Jackson, *Classical Electrodynamics*

\[ \frac{3\varepsilon_b}{\Delta \varepsilon + 3\varepsilon_b} E_0 \]

\[ E = E_0 \text{ for small } \Delta \varepsilon \text{ only !} \]

Just plug $\Delta \varepsilon$ in perturbation formulas fails for large $\Delta \varepsilon$. 
**assumptions**: the field distribution in the plane in the immediate vicinity of the slit becomes independent of the metallic permittivity (as $|\varepsilon|\to\infty$). (the particle dimensions are larger than the skin depth)
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The initial launching of the SPP changes: $H_{SP} \propto |\varepsilon|^{-1/2}$

$H'_{inc} = H_{inc}$

$\delta_1 \propto \lambda (\varepsilon')^{1/2}$
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   - definition of scattering coefficients for the quasi-CW
   - dual wave picture microscopic model
What is due to SPP in the EOT

Transmittance vs. \( \lambda/a \)

- **RCWA**
- **SPP model**

- \( a=0.68 \, \mu m \)
- \( a=0.94 \, \mu m \)
- \( a=2.92 \, \mu m \)

\( \theta=0^\circ \)
Dual wave picture

$\approx x - m \exp(i k_0 x) = \exp(i k_{\text{SP}} x)$

$\lambda = 940 \text{ nm}$

Dual wave picture

\[ \text{total field} \]

\[ \text{SPP} = \exp(i k_{\text{SP}} x) \]

\[ \lambda = 940 \text{ nm} \]

Dual wave picture

Young's slit experiment

Young's slit experiment

S. Ravets et al., JOSA B 26, B28 (2009).

$\theta$ (°) in air

$\lambda=850 \text{ nm}$
Computational results

Far-field intensity (a.u.)

\( \lambda = 0.6 \, \mu m \)

\( \lambda = 1 \, \mu m \)

\( \lambda = 3 \, \mu m \)

\( \lambda = 10 \, \mu m \)

PC

S. Ravets et al., JOSA B 26, B28 (2009).

\( \theta (°) \)

\( \lambda \)

SPP mainly

Quasi-CW & SPP

Quasi-CW mainly

Quasi-CW only
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\[ E_s, H_s = \text{scattered field} \]
\[ E \text{ actual field} \]
\[ \nabla \times E_s = j \omega \mu_0 H_s \]
\[ \nabla \times H_s = -j \omega \varepsilon(r) E_s - j \omega \Delta \varepsilon E \]

1/Hypothesis: the sub-\( \lambda \) indentation can be replaced by an effective dipole \( p = p_x x + p_y y \). When is it reliable?

Polarizability tensor
\[
p_x = (\alpha_{xx} E_{x,\text{inc}} + \alpha_{xz} E_{z,\text{inc}}) x \\
p_z = (\alpha_{zx} E_{x,\text{inc}} + \alpha_{zz} E_{z,\text{inc}}) z
\]
\[ \nabla \times E_s = j \omega \mu_0 H_s \]
\[ \nabla \times H_s = -j \omega \varepsilon(r) E_s - j \omega \Delta \varepsilon E \]

1/Hypothesis: the sub-\( \lambda \) indentation can be replaced by an effective dipole \( p = p_x x + p_y y \).

2/The effective dipoles \( p_x \) and \( p_y \) are unknown. They probably depend on many parameters especially for sub-\( \lambda \) indentation that are not much smaller than \( \lambda \) (such as resonant grooves).
\( E_s, H_s = \text{scattered field} \)
\( E \text{ actual field} \)

\[ \begin{align*}
\nabla \times E_s &= j \omega \mu_0 H_s \\
\nabla \times H_s &= -j \omega \varepsilon(r)E_s - j \omega \Delta \varepsilon E
\end{align*} \]

1/\text{Hypothesis} : the sub-\( \lambda \) indentation can be replaced by an effective dipole \( p = p_x x + p_y y \).

2/\text{The effective dipoles} \( p_x \) and \( p_y \) are unknown.

3/ We solve Maxwell's equation for both dipole source
\( \nabla \times E = j \omega \mu_0 H \)
\( \nabla \times H = -j \omega \varepsilon(r)E + (E_{s,x} x + E_{s,y} y) \delta(x,y) \)
The bad scenario
The actual scenario

$\lambda = 800$ nm, gold

The two dipole sources approximately generate the same field.
Es, Hs = scattered field
E actual field
\[
\begin{align*}
\nabla \times E_s &= j\omega \mu_0 H_s \\
\nabla \times H_s &= -j\omega \varepsilon(r) E_s - j\omega \Delta \varepsilon E
\end{align*}
\]

1/Hypothesis: the sub-\(\lambda\) indentation can be replaced by an effective dipole \(p = p_x x + p_y y\).

2/The effective dipoles \(p_x\) and \(p_y\) are unknown.

3/The field scattered (in the vicinity of the surface for a given frequency) has always the same shape:

\[
\Phi(x,y) = [\alpha_{SP} E_{inc}] \Phi_{SP}(x,y) + [\alpha_{CW} E_{inc}] \Phi_{CW}(x,y)
\]
Analytical expression for the quasi-CW

Cauchy theorem

\[ H = H_{SP} + H_{CW} \]

\[ H_{SP} = \frac{k_{SP}^2 \varepsilon_{SP} \varepsilon_{m}}{k_0^2 \varepsilon_m - \varepsilon_d} \exp(ik_{SP}x) \]

\[ H_{CW} = \text{Integral over a single real variable} \]

Dominated by the branch-point singularity

A single pole singularity: SPP contribution

Two Branch-cut singularities: \( \Gamma_m \) and \( \Gamma_d \)
Maths have been initially developed for the transmission theory in wireless telegraphy with a Hertzian dipole radiating over the earth


5/W. Dai & C. Soukoulis, "Theoretical analysis of the surface wave along a metal-dielectric interface", PRB accepted for publication (private communication).
6/L. Martin Moreno, F. Garcia-Vidal, SPP4 proceedings 2009.
Analytical expression for the quasi-CW

Maxwell's equations
\[ \nabla \times \mathbf{E} = j \omega \mu_0 \mathbf{H} \]
\[ \nabla \times \mathbf{H} = -j \omega \varepsilon(r) \mathbf{E} + (E_{s,x}x + E_{s,y}y) \delta(x,y) \]

Analytical solution

\[ \left[ \frac{\chi_m}{\varepsilon_m} E_{s,x} + \frac{n_d}{\varepsilon_d} \varepsilon_s E_{s,y} \right] \times \begin{pmatrix} H_{z,CW} \\ E_{x,CW} \\ E_{y,CW} \end{pmatrix} \]

\( \varepsilon_s \) is either \( \varepsilon_d \) or \( \varepsilon_m \), whether the Dirac source is located in the dielectric material or in the metallic medium.

Under the Hypothesis that

- \( |\varepsilon_m| >> \varepsilon_d \)
- \( z < \lambda \)
- \( x > \lambda/2\pi \)
quasi-CW for gold at $\lambda=940$ nm
Intrinsic properties of quasi-CW

Hypothesis
- $z < \lambda$
- $x > \lambda / 2\pi$

$F(x)$ is a slowly-varying envelop

$[H_{y,0}, E_{x,0}, E_{z,0}]$ is the normalized field associated to the limit case of the reflection of a plane-wave at grazing incidence

Grazing plane-wave field

\[
\begin{pmatrix}
H_{y,CW} \\
E_{x,CW} \\
E_{z,CW}
\end{pmatrix}
= F(x)
\begin{pmatrix}
H_{y,0} \\
E_{x,0} \\
E_{z,0}
\end{pmatrix}
\]

- linear z-dependence for \( z < \lambda \)
- Main fields are almost null for \( z \approx (\lambda /2\pi) |\varepsilon_m|^{1/2} \)
- nearly an \( \exp(ik_0x) \) x-dependence for \( x >> \lambda \)

Closed-form expression for $F(x)$

$|F(x)|$ (a.u.)

$\propto 1/\sqrt{x}$

$\propto 1/\sqrt{x^3}$

silver @ $\lambda=1 \mu m$

Closed-form expression for $F(x)$

Highly accurate form for any $x$

$$F(x) = \exp(ik_0 x) \ W[2\pi(n_{SP}-n_d)x/\lambda] \ (x/\lambda)^{3/2} \text{ with } W(t) \text{ an Erf-like function}$$

Highly accurate for $x < 10\lambda$

$$F(x) = \exp(ik_0 x) \ (x/\lambda)^{-m}$$

$m$ varies from 0.9 in the visible to 0.5 in the far IR

$m=0.83$ for silver @ 852 nm
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Scaling law

\( \lambda = 0.633 \, \mu m \)

\( \lambda = 1 \, \mu m \)

\( \lambda = 3 \, \mu m \)

\( \lambda = 9 \, \mu m \)

|H| (a.u.) vs. \( x/\lambda \)

\( H_{SP} \)

\( H_{CW} \)

\((x/\lambda)^{-1/2} \) (PC)

(result for silver)

Scaling law

\[ \lambda = 0.633 \, \mu m \]

\[ \lambda = 1 \, \mu m \]

\[ \lambda = 3 \, \mu m \]

\[ \lambda = 9 \, \mu m \]

1. The emblematic example of the EOT
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   - a microscopic pure-SPP model of the EOT

2. SPP generation by 1D sub-$\lambda$ indentation
   - rigorous calculation (orthogonality relationship)
   - the important example of slit
   - scaling law with the wavelength

3. The quasi-cylindrical wave
   - the importance of the quasi-CW
   - definition & properties
   - scaling law with the wavelength
   - experimental evidence

4. Microscopic theory of sub-$\lambda$ surfaces
   - definition of scattering coefficients for the quasi-CW
   - dual wave picture microscopic model
Slit-Groove experiment

Fall off for $d < 5\lambda$

$\lambda = 852$ nm

$|S/S_0|^2$

$|S|^2$

$|S_0|^2$


promote an other model than SPP & quasi-CW (CDEW model)

frequency = $1.05 k_0$

$k_{SP} = k_0 [1 - 1/(2\varepsilon_{Ag})] \approx 1.01k_0$
Near field validation

G. JULIE, V. MATHET
IEF, Orsay

L. AIGOUY
ESPCI, Paris

λ=975 nm

2 µm

experiment

2 µm

computation

slit

slit

TM
Field recorded on the surface

\[ \lambda = 974 \text{ nm} \]
\[ E_z = A_{SP} \sin(k_{SP}x) + A_c \left( \frac{\exp(ik_0x)}{(x+d)^m} - \frac{\exp(-ik_0x)}{(x-d)^m} \right) \]

- **Standing SPP**
- **Right-traveling cylindrical wave**
- **Left-traveling cylindrical wave**

**Fitted parameters**
- \( A_{SP} \) (real)
- \( A_c \) (complex)

(m=0.5)
L. Aigouy et al., PRL 98, 153902 (2007)
A direct observation?

If one controls the two beam intensity and phase (or the separation distance) so that there is no SPP generated on the right side, do I observe a pure quasi-CW on the right side of the doublet?
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Defining scattering coefficients for CWs

The SPP is a normal mode

⇒ elastic scattering coefficients like the SPP transmission may be easily defined.

You may also define inelastic scattering coefficients with other modes like the radiated plane waves

You may use mode orthogonality and reciprocity relationships.
**CW-to-SPP cross-conversion**

![Diagram showing CW-to-SPP cross-conversion](image)
Other scattering coefficients

\[ \beta_{CW} = \beta_{SP} \]
\[ \alpha_{CW} = \alpha_{SP} \]

Cross-conversion scattering coefficients

Ansatz: THE SCATTERED FIELDS ARE IDENTICAL.
(if the two waves are normalized so that they have identical amplitudes at the slit)

Scaling law for $r_c$ and $t_c$

\[ |r_c|^2 \text{ & } |t_c|^2 \]

\[ |\varepsilon_m|^{-1} \]

\[ \lambda (\mu m) \]

\[ \text{Re}(r_c) \quad \text{Im}(r_c) \quad \text{Re}(t_c) \quad \text{Im}(t_c) \]

\[ \lambda (\mu m) \]

\[ 10^{-4} \quad 10^0 \quad 10^1 \]

\[ 10^{-2} \quad 10^{-1} \]
Other scattering coefficients

\[ \beta_{CW} = \beta_{SP} \]
\[ \alpha_{CW} = \alpha_{SP} \]

Mixed SPP-CW model for the extraordinary optical transmission

pure SPP model

\[
\begin{align*}
    t_A &= t + \frac{2\alpha \beta}{u^{-1} - (\rho + \tau)} \\
    r_A &= \frac{2\alpha^2}{u^{-1} - (\rho + \tau)}
\end{align*}
\]

mixed SPP-CW model

\[
\begin{align*}
    t_A &= t + \frac{2\alpha \beta}{(P^{-1}+1) - (\rho + \tau)} \\
    r_A &= \frac{2\alpha^2}{(P^{-1}+1) - (\rho + \tau)} \\
    P &= \frac{1}{(u^{-1} - 1)} + \sum_{n=1}^{\infty} H_{CW}(na)
\end{align*}
\]

H. Liu et al. (submitted)
Pure SPP-model prediction of the EOT

Mixed SPP-CW model for the extraordinary optical transmission

\[ \theta = 0^\circ \]
SPP+quasi-CW coupled-mode equations

It is not necessary to be periodic
It is not necessary to deal with the same indentations
Conclusion

Two different microscopic waves, the SPP mode and the quasi-CW, are at the essence of the rich physics of metallic sub-\(\lambda\) surfaces

Their relative weights strongly vary with the metal permittivity

They exchange their energy through a cross-conversion process, whose efficiency scales as \(|\varepsilon_m|^{-1}\)

The local SPP elastic or inelastic scattering coefficients are essential to understand the optical properties, since they apply to both the SPP and the quasi-CW
SPP, quasi-CW, C-SPP, quasi-SP

Quasi-Spherical Wave

Cylindrical SPP

Quasi-SW
Cylindrical SPP

$E \text{ (normal)}$

$y/\lambda$

gold @ $\lambda=800 \text{ nm}$